Magnetic Attitude Control of a Flexible Satellite

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Nomenclature

$\mathbf{1}$ = identity matrix

$\mathbf{1}_n$ = unit vector defined by $\{\mathbf{1}_n\}_i = \begin{cases} 0, & i \neq n \\ 1, & i = n. \end{cases}$

$\mathbf{a}^\times$ = skew-symmetric cross-product matrix associated with $\mathbf{a} \in \mathbb{R}^3$

$\mathbf{b}_b$ = Earth's magnetic field vector in $\mathcal{F}_b$ coordinates (T)

d_{ij} = viscous damping constant for panel i about axis j (Nms/rad)

k_{ij} = spring stiffness for panel i about axis j (Nm/rad)

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$\mathbf{J}_b$ = moment of inertia matrix for spacecraft central body (kg·m^2)

$\mathbf{J}_r$ = moment of inertia matrix for entire spacecraft in undeformed state (kg·m^2)

$\mathbf{h}_w$ = wheel bias momentum vector (Nms)

h_w = wheel bias momentum (Nms)

$\mathbf{m}$ = magnetic torquer dipole vector (A·m^2)

T = orbital period (s)

u_w = desired control torque to be applied to spacecraft by the wheel (Nm)

$\mathbf{u}_m$ = desired control torque to be applied to spacecraft by the magnetic torquers (Nm)

$\delta\boldsymbol{\omega}$ = angular velocity relative to orbiting frame (rad/s)

$[\boldsymbol{\epsilon}, \epsilon_4]$ = quaternion representation of attitude relative to orbiting frame

ω_o = orbital angular rate (rad/s)

$\boldsymbol{\omega}$ = inertial angular velocity (rad/s)

θ_j^{ib} = i^{th} panel rotation about axis j (rad)

$\tilde{\boldsymbol{\theta}}^{ba}$ = $\left[\theta_x^{ib}, \theta_y^{ib}\right]^T$ (rad)

$\boldsymbol{\tau}$ = total applied control torque (Nm)

$\boldsymbol{\tau}_m$ = magnetic torque (Nm)

I. Introduction

This note presents a study on attitude control of a flexible satellite using three mutually perpendicular magnetic torque rods (magnetorquers) and a single reaction wheel. The research is motivated by JC2Sat, a proposed satellite formation flying mission using differential drag as the means of formation control [1]. To increase or decrease atmospheric drag for each satellite, pitch attitude maneuvers are performed to increase or decrease the satellite frontal (drag) area. The purpose of the momentum wheel is to gyroscopically stabilize the pitch axis and perform rapid pitch maneuvers. Roll and yaw stabilization is provided by magnetic torquers. To make formation control using differential drag feasible, this type of satellite generally has relatively small mass and large drag panels (to increase the ballistic coefficient). This can result in significant satellite structural flexibility, which could degrade the performance of the attitude control system. To the authors' best knowledge, simultaneous attitude control and vibration suppression using magnetic actuation has not been treated in the literature previously.

The motivation for the study presented in this note is to see whether it is possible to actively suppress structural vibrations when magnetic actuation is used as the means for attitude control. The answer to this question is not immediately obvious for two reasons. First, magnetic actuation has inherently low control authority. Second, spacecraft with magnetic control are instantaneous underactuated. This is due to the well-known fact that magnetic torquers can only generate torques perpendicular to the local Earth magnetic field vector [2]. It is the variation of the local Earth's magnetic field within an orbit that provides on-average controllability when magnetic actuation is used [2]. A recent survey outlines several methods used in magnetic attitude control [3]. The different methods can be grouped into linear and non-linear approaches.

The most common method among the linear control design techniques is to take advantage of the (quasi) periodic nature of the Earth's magnetic field. This involves using the linearized state-space model of the system and solving a Periodic Riccati Equation (PRE) to get an optimal time-periodic set of control gains. Using periodic control theory, it has been shown that the linearized closed-loop system is asymptotically stable [4]. A similar approach has also been used for disturbance torque attenuation [5]. In [6], an infinite horizon, a finite horizon, and a time-invariant controller are proposed and compared. It is found for circular orbits, the finite horizon controller performs much better than the infinite horizon and time-invariant controllers. Another time-invariant controller was proposed in [7] which uses a constant gain matrix that approximates the solution to the PRE. It however, does not guarantee asymptotic stability, unlike the time-varying solution.

Among the non-linear control design techniques, a number of works [8–10] again use the periodicity assumption of the Earth's magnetic field and use Krasovskii-LaSalle type arguments to prove local asymptotic stability. The resulting controllers have the challenges of being time-varying. A different approach in [11] and [12], which does not make any periodicity assumptions, provides a time-invariant proportional-derivative (PD) type control law. This has been done for both an inertially pointing [11] and Earth pointing [12] satellite. A sufficient condition for stability in [11] and [12] is that the control gains must be less than an upper bound. However, there is no analytical way of determining the upper bound. As a consequence numerical simulations must be performed to verify closed-loop stability. In [13], it is shown that if a minimum level of an independent 3-axis control system, such as reaction wheels, is used, the gain limitation in [11] and [12] can be removed. The methods in the works mentioned here only treat magnetic attitude control of rigid spacecraft. To the authors' best knowledge, the only published work on using magnetic attitude control of flexible satellites is [14], which is an extension of the work in [11, 12]. Similar to [11, 12], it is shown in [14] that if the control gains are below some upper bound (which cannot be determined analytically), asymptotic stability is guaranteed even in the presence of perturbations from flexible appendages. It does not, however, attempt to actively suppress the flexible vibrations.

In this note, we propose two controllers that perform simultaneous attitude control and active vibration suppression using magnetic actuation. The first controller is time-invariant, and the second is periodic. Both controllers are based on Linear Quadratic Regulator (LQR) theory. The time-invariant controller commands a desired magnetic control torque. The control torque is implemented magnetically by projecting it onto the plane perpendicular to the local Earth magnetic field. The second controller directly commands magnetic torquer dipole moment as the control input. Due to the approximate periodic nature of the local Earth magnetic field as seen on orbit, the resulting controller is periodic. The periodic controller takes inspiration from those presented in [4, 5]. A significant differentiating feature in this work from [4, 5] is that unlike in those papers, the flexible dynamics are explicitly incorporated into the control design. The advantage of the time-invariant controller over the periodic controller is that it has significantly less computational storage requirements.

II. Satellite Model

The type of satellite under consideration has large drag panels attached to the satellite body using tape springs. Tape springs provide a lightweight and simple deployment mechanism, but once deployed can be highly flexible. For small deflections, the tape springs can be treated as torsion springs [15]. The satellite model is treated as a central body with rigid drag panels attached by torsion springs such that there is both flapping and torsional vibrations for both panels as shown

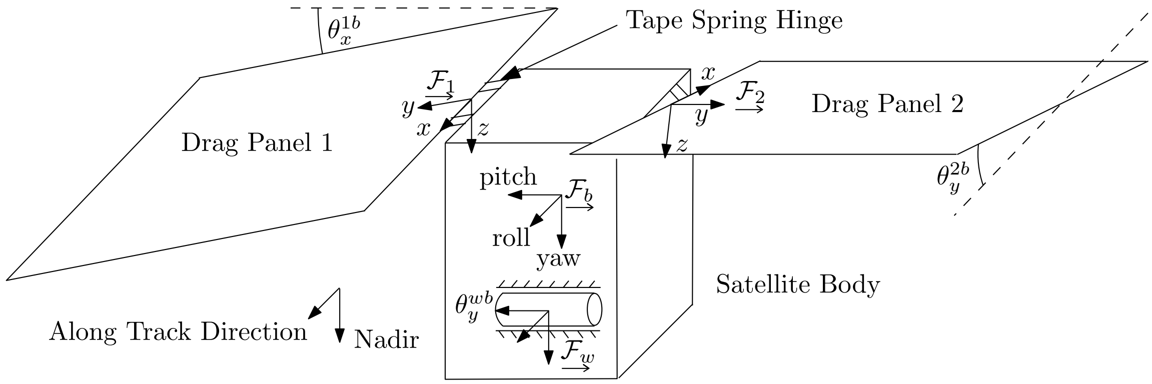

Fig. 1 Satellite showing hinge bending about x and y axes.

in Fig. 1. The flapping and torsional vibrations are about the x and y axes of the panel frames, with angles θ_x^{ib} and θ_y^{ib} respectively, for panels $i = 1, 2$. The satellite is assumed to be in a circular orbit, and the commanded attitude is nominally nadir-pointing, with commanded pitch maneuvers every half orbit (see Section IV for more details). Attitude control is provided by three mutually perpendicular magnetic torquers and a single momentum wheel aligned with the satellite's pitch axis.

The full set of non-linear equations of motion can be found in [16]. These equations are not used for the design of the control laws (the appropriate models are given in Section III.A.), but they are used in section IV for the numerical simulation of the closed-loop system.

III. Controller Design

Four controllers are compared. All are linear, two are time-invariant and two are periodic. The simplest is a time-invariant controller which neglects flexibility (it assumes the satellite to be rigid). The other, more complex, time-invariant controller attempts to actively suppresses the panel vibrations by using an observer to estimate the unmeasured panel states. Both have the advantage of only needing a single constant gain matrix stored on board the satellite. The time-varying controllers take advantage of the roughly periodic nature of the Earth's magnetic field to determine a periodic gain matrix. The drawback is that the large amount of on board data storage required to save gain matrices over time may make the controller infeasible to implement. However, the periodic nature of the gains makes it possible to replace them using Fourier series approximations

[17], greatly reducing the date storage requirements. As for the time-invariant case, one of the periodic controllers neglects flexibility, and the other actively suppresses vibrations. The controllers will be referred to as the time-invariant rigid (TIR), time-invariant vibration suppression (TIVS), periodic rigid (PR) and periodic vibration suppression (PVS), respectively.

A. Linear Models Used For Control Design

This section presents the linear state space models used for each of the control law designs.

It is assumed that both the attitude $\boldsymbol{\epsilon}$ and angular velocity $\delta\boldsymbol{\omega}$, relative to the orbiting frame, are available as measurements. As explained in more detail in Section IV, the commanded attitude is piecewise constant relative to an orbiting frame, with changes occurring each half orbit. Assuming a momentum management scheme for the wheel, the wheel bias momentum h_w will stay near the set-point. The only possibly significant deviations of h_w occur during maneuvers. However, these maneuvers are short, so the deviation of h_w is temporary. Therefore, for the purposes of the control design it is assumed that h_w is constant.

Linear Model for TIR Control Design

When flexibility is neglected, the linear model is given by a linearization of Euler's equation together with the quaternion kinematics about the nadir-pointing attitude [18],

$$\begin{aligned}
\dot{\mathbf{x}} &= \underbrace{\begin{bmatrix} \mathbf{O} & \frac{1}{2}\cdot\mathbf{1} \\ \mathbf{J}_r^{-1}\mathbf{A}_{21} & \mathbf{J}_r^{-1}\mathbf{A}_{22} \end{bmatrix}}_{\mathbf{A}_R} \underbrace{\begin{bmatrix} \boldsymbol{\epsilon} \\ \delta\boldsymbol{\omega} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \mathbf{O} \\ \mathbf{J}_r^{-1}\begin{bmatrix} \mathbf{1}_2 & \mathbf{1} \end{bmatrix} \end{bmatrix}}_{\mathbf{B}_{TIR}} \underbrace{\begin{bmatrix} u_w \\ \mathbf{u}_m \end{bmatrix}}_{\mathbf{u}_{TI}}, \qquad (1) \\
\mathbf{y} &= \underbrace{\begin{bmatrix} \mathbf{1} & \mathbf{O} \\ \mathbf{O} & \mathbf{1} \end{bmatrix}}_{\mathbf{H}_R} \begin{bmatrix} \boldsymbol{\epsilon} \\ \delta\boldsymbol{\omega} \end{bmatrix}
\end{aligned}$$

where

$$\mathbf{A}_{21} = \mathrm{diag}\{a, 0, a\}, \qquad (2)$$

with $a = 2\left(h_w\omega_0 - (J_{ry} - J_{rz})\omega_0^2\right)$, and

$$\mathbf{A}_{22} = \begin{bmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ -b & 0 & 0 \end{bmatrix}, \tag{3}$$

with $b = h_w + \omega_0(J_{rx} + J_{rz} - J_{ry})$. Note that J_{rx}, J_{ry} and J_{rz} are the moment of inertias about the x, y and z axes respectively, assuming a principal axes frame for $\mathbf{J}_r$. The control law design will provide laws for the desired torques, u_w and $\mathbf{u}_m$. The actual wheel torque, $\boldsymbol{\tau}_w = [0\ u_w\ 0]$, is the same as the desired torque. However, the magnetic torquers cannot in general deliver $\mathbf{u}_m$, since the actual torque $\boldsymbol{\tau}_m$ is always perpendicular to $\mathbf{b}_b$, as evidenced by [18]

$$\boldsymbol{\tau}_m = \mathbf{m}^\times \mathbf{b}_b. \tag{4}$$

Consequently, we project the desired magnetic torque $\mathbf{u}_m$ onto the plane perpendicular to the local Earth's magnetic field, $\mathbf{b}_b$. The commanded magnetic torquer dipole moment $\mathbf{m}$ which realizes this torque is computed from [2]

$$\mathbf{m} = \|\mathbf{b}_b\|^{-2}\, \mathbf{b}_b^\times \mathbf{u}_m, \tag{5}$$

where $\|\cdot\|$ denotes the Euclidean norm.

Linear Model for PR Control Design

For the PR controller, we re-use the TIR model in (1), replacing the control input $\mathbf{u}_m$ with the magnetic torquer dipole moment $\mathbf{m}$. Therefore, using (4), the linear PR model is the same as (1), with $\mathbf{B}_{TIR}$ and $\mathbf{u}_{TI}$ replaced by

$$\mathbf{B}_{PR}(t) = \begin{bmatrix} \mathbf{O} \\ \mathbf{J}_r^{-1}\begin{bmatrix} \mathbf{1}_2 & -\hat{\mathbf{b}}_b^\times \end{bmatrix} \end{bmatrix}, \quad \mathbf{u}_P = \begin{bmatrix} u_w \\ \hat{\mathbf{m}} \end{bmatrix} \tag{6}$$

where $\hat{\mathbf{b}}_b = \frac{\mathbf{b}_b}{\|\mathbf{b}_b\|}$ and $\mathbf{m} = \|\mathbf{b}_b\|\, \hat{\mathbf{m}}$.

Linear Models for TIVS and PVS Control Designs

For the TIVS and PVS control designs, the panel states are included such that the state vector is

$\mathbf{x}^\mathsf{T} = [\boldsymbol{\epsilon}^T\ \tilde{\boldsymbol{\theta}}^{1bT}\ \tilde{\boldsymbol{\theta}}^{2bT}\ \ \delta\boldsymbol{\omega}^T\ \dot{\tilde{\boldsymbol{\theta}}}^{1bT}\ \dot{\tilde{\boldsymbol{\theta}}}^{2bT}]$. The linear model used for the TIVS controller is

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{O} & \bar{\mathbf{A}}_{12} \\ \tilde{\mathbf{M}}^{-1}\mathbf{A}_{21} & \tilde{\mathbf{M}}^{-1}\bar{\mathbf{A}}_{22} \end{bmatrix}}_{\mathbf{A}_{VS}} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{O} \\ \tilde{\mathbf{M}}^{-1} \begin{bmatrix} \mathbf{1}_2\ \mathbf{1} \\ \mathbf{O} \end{bmatrix} \end{bmatrix}}_{\mathbf{B}_{TIVS}} \mathbf{u}_{TI}, \tag{7}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} \mathbf{1} & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{1} & \mathbf{O} \end{bmatrix}}_{\mathbf{H}_{VS}} \mathbf{x}$$

where $\bar{\mathbf{A}}_{12} = \text{diag}\{0.5\ 0.5\ 0.5\ 1\ 1\ 1\ 1\}$. $\bar{\mathbf{A}}_{21} = \text{diag}\{\mathbf{A}_{21}\ \ -k_{1x}\ \ -k_{1y}\ \ -k_{2x}\ \ -k_{2y}\}$, and $\bar{\mathbf{A}}_{22} = \text{diag}\{\mathbf{A}_{22}\ \ -d_{1x}\ \ -d_{1y}\ \ -d_{2x}\ \ -d_{2y}\}$. Note that $\mathbf{A}_{21}$ and $\mathbf{A}_{22}$ are given in (2) and (3) respectively.

The linear model for the PVS controller is the same as the TIVS model (7), with $\mathbf{B}_{TIVS}$ replaced by

$$\mathbf{B}_{PVS}(t) = \begin{bmatrix} \mathbf{O} \\ \tilde{\mathbf{M}}^{-1} \begin{bmatrix} \mathbf{1}_2\ \ -\hat{\mathbf{b}}_b^\times \\ \mathbf{O} \end{bmatrix} \end{bmatrix}, \tag{8}$$

and $\mathbf{u}_{TI}$ replaced by $\mathbf{u}_P$ in (6).

B. Control Design

All four controllers (TIR, PR, TIVS and PVS) are designed using the Linear Quadratic Regulator (LQR) framework [19] together with a state estimator based on a steady-state Kalman filter [19].

All systems in Section III.A. take the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(t)\mathbf{u},$$

$$\mathbf{y} = \mathbf{H}\mathbf{x},$$

where $\mathbf{B}$ is constant in the time-invariant case, and T-periodic in the periodic case. The control laws take the form

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^\mathsf{T}(t)\mathbf{P}(t)\hat{\mathbf{x}}_e \tag{9}$$

$$\dot{\hat{\mathbf{x}}}_e = \mathbf{A}\hat{\mathbf{x}}_e + \mathbf{B}\boldsymbol{\tau} + \mathbf{L}(\mathbf{H}\hat{\mathbf{x}}_e - \mathbf{H}\mathbf{x}_e) \tag{10}$$

where $\mathbf{L}$ is the filter gain, $\mathbf{P}(t)$, is the symmetric positive-definite steady-state solution of the Riccati Equation [20]

$$-\dot{\mathbf{P}}(t) = \mathbf{P}(t)\mathbf{A} + \mathbf{A}^{\mathsf{T}}\mathbf{P}(t) - \mathbf{P}(t)\mathbf{B}(t)\mathbf{R}^{-1}\mathbf{B}^{\mathsf{T}}(t)\mathbf{P}(t) + \mathbf{Q}, \tag{11}$$

and $\mathbf{Q} \geq \mathbf{0}$ and $\mathbf{R} > \mathbf{0}$ are LQR weighting matrices. Note that $\mathbf{P}$ is constant in the time-invariant case, and T-periodic in the periodic case. The vector $\mathbf{x}_e$ is the difference between the actual states, $\mathbf{x}$, and the desired ones and $\hat{\mathbf{x}}_e$ is the estimate of $\mathbf{x}_e$. The distinction between $\mathbf{x}_e$ and $\mathbf{x}$ is made because during the half of the orbit where the desired attitude is pitched forward (see Section II and IV), $\mathbf{x}_e$ and $\mathbf{x}$ are different. The filter gain is chosen according to $\mathbf{L}$ is given by

$$\mathbf{L} = -\mathbf{P}_e\mathbf{H}^{\mathsf{T}}\mathbf{Q}_v^{-1}$$

where $\mathbf{P}_e > \mathbf{0}$ is the solution to the Algebraic Riccati Equation,

$$\mathbf{P}_e\mathbf{A}^{\mathsf{T}} + \mathbf{A}\mathbf{P}_e - \mathbf{P}_e\mathbf{H}^{\mathsf{T}}\mathbf{Q}_v^{-1}\mathbf{H}\mathbf{P}_e + \mathbf{Q}_w = \mathbf{O}$$

where $\mathbf{Q}_w > \mathbf{0}$ and $\mathbf{Q}_v > \mathbf{0}$ represent the covariances of the process noise and measurement noise respectively in a steady-state Kalman filter [19], but are tuned manually using simulations in the simulation in Section IV.

If $(\mathbf{A}, \mathbf{B})$ are stabilizable and $(\mathbf{Q}^{\frac{1}{2}}, \mathbf{A})$ are detectable, and if $(\mathbf{H}, \mathbf{A})$ are observable then the control by Eq. (9) and (10) are stabilizing for the linearized system [19]. However, these stability guarantees are lost in the time-invariant cases when the control law is implemented via (4). That being said, there are several specific cases in the literature of magnetic attitude control using control implementations of the form (4), for which analytical stability proofs have been provided (see [1, 11, 12, 14, 21, 22]). In the absence of an analytical stability proof for the time-invariant controllers in this note, we provide Monte-Carlo type analysis with numerical simulations in Section IV to demonstrate stability.

IV. Simulation

In this section, the proposed controllers are evaluated numerically using the exact non-linear dynamic model for the JC2Sat satellites, which may be found in [16]. The numerical simulation and control parameters may also be found in [16].

As explained in Section I, JC2Sat is a proposed formation flying mission using differential drag as the means for formation control. To mimic steady-state formation maintenance, the commanded pitch angle is zero for half of each orbit, and 60^o for the other half of each orbit. To reduce the sudden change in input to the control system, a smoothing function is used when the commanded pitch angle changes. The function spans 0.5% of the orbit (approximately 30 seconds). Saturation constraints for the magnetic torquers are enforced. Gaussian noise with a standard deviation of 0.5° is added to the attitude measurements. Arbitrary 5% adjustments are made to the satellite moment of inertia, the panel spring constants, as well as the applied control torque. This is to model uncertainties in physical satellite parameters as well as actuator output scaling and misalignment errors. Gravity gradient and residual magnetic dipole disturbance torques are included. The magnetic field model used in both the simulation and in determining the PRE solutions is a Tilted Dipole Model [2].

Tenth-order Fourier series are used to approximate $\mathbf{P}(t)$ in the periodic cases.

For each controller type, a set of Monte-Carlo type simulations are performed with randomly generated initial conditions. The purposes of this are two fold: first, to demonstrate stability of the controllers, in particular the TIR and TIVS controllers for which there are no analytical guarantees, and second, to compare the performances of all four controllers across a broad range of initial conditions. The initial attitude error, angular velocity and panel deflections are randomly generated from a zero-mean normal distribution, with standard deviations of 3 deg (Euler angle), 0.6 deg/s (angular velocity) and 7 deg (panel deflection) respectively. Forty simulations are performed for each controller type. It is important to note that while the initial conditions for each controller type are randomly generated, each set of simulations (per controller type) are started with the same random number generator seed value. This means that each controller type has the same set of forty initial conditions, making a comparison among them fair.

A typical set simulation results (with non-zero initial conditions) are shown in Figure 2, where the TIR and TIVS controllers are compared for a four orbit simulation. The results indicate that vibrations are mainly induced by non-zero initial conditions and the twice per orbit pitch maneuvers. It can be seen that the TIVS controller is able to stabilize the attitude slightly faster as well as have significantly smaller panel deflections. The results for the PR and PVS controllers are similar, and are therefore not shown. It can also be seen that the closed-loop system converges within one and a half orbits. Therefore, all subsequent simulations are performed for two orbits.

A summary of the controller performances may be found in Table 1. Both the mean and peak (worst case) performances across each set of forty simulations are presented. First of all, it is clear from the peak performance values that all controllers are stabilizing. As seen from both the mean and peak performance values in Table 1, the TIVS and PVS controllers (which actively suppress vibrations) consistently outperform the TIR and PR controllers (which neglect vibrations) in terms of both attitude regulation as well as vibration suppression. However, this comes at a cost of slightly increased magnetic torquer activity, which is to be expected. This confirms it is indeed possible to actively suppress spacecraft vibrations using magnetic actuation. On the other hand, the periodic controllers, PR and PVS (which incorporate the Earth magnetic field in the control design), only slightly outperform the time-invariant controllers, TIR and TIVS (which do not incorporate the Earth magnetic field in the control design). Given the increased computational requirements of storing periodic gains on-board, this suggests that use of the time-invariant controllers is sufficient for practical purposes.

As mentioned, one of the main causes of vibration is the twice per orbit pitch maneuvers. A possible approach to further reduce the induced vibrations is input shaping of the pitch command [23]. This has been demonstrated to be quite effective in other attitude control configurations when used in conjunction with an active vibration suppression scheme [24].

V. Conclusions

Time-invariant and periodic controllers have been proposed for simultaneous attitude control and vibration suppression for a flexible bias-momentum spacecraft using magnetic actuators.

The time-invariant controller projects a stabilizing control torque onto the plane perpendicular to the local Earth magnetic field for implementation by the magnetic torquers. By performing this projection, analytical stability guarantees are lost. However, stability is demonstrated by Monte-Carlo numerical simulation. The periodic controller directly provides the magnetic torquer dipole

Mean	TIR	PR	TIVS	PVS
$\boldsymbol{\theta}$ rms error (°)	1.20	1.03	0.779	0.707
$\boldsymbol{\omega}$ rms error (°/s)	0.0748	0.0664	0.0505	0.0452
$\tilde{\boldsymbol{\theta}}^{1b}$ rms (°)	0.476	0.468	0.260	0.247
$\tilde{\boldsymbol{\theta}}^{2b}$ rms (°)	1.01	0.994	0.502	0.474
Average $\|\mathbf{m}\|_1$ (A·m^2)	0.397	0.415	0.466	0.506
Average $\|u_w\|$ (N·m)	10.6×10^{-5}	10.5×10^{-5}	6.36×10^{-5}	6.46×10^{-5}
Peak	**TIR**	**PR**	**TIVS**	**PVS**
$\boldsymbol{\theta}$ rms error (°)	1.70	1.60	1.19	1.10
$\boldsymbol{\omega}$ rms error (°/s)	0.132	0.121	0.0926	0.0844
$\tilde{\boldsymbol{\theta}}^{1b}$ rms (°)	0.693	0.756	0.492	0.474
$\tilde{\boldsymbol{\theta}}^{2b}$ rms (°)	1.47	1.64	0.947	0.907
Average $\|\mathbf{m}\|_1$ (A·m^2)	0.693	0.674	0.712	0.760
Average $\|u_w\|$ (N·m)	11.5×10^{-5}	11.6×10^{-5}	7.35×10^{-5}	7.45×10^{-5}

Table 1 Monte-Carlo performance summary for all four controllers.

moment as the control input. Analytical stability guarantees can be made in this case.

The performances of these two proposed controllers have been compared numerically with similar time-invariant and periodic controllers, which neglect the satellite flexibility. It has been shown that despite the inherently low control authority and instantaneous underactuation with magnetic control, the proposed controllers do significantly reduce the induced vibrations, and provide more accurate attitude control (compared to the controllers that neglect flexibility). It was found that the periodic controller performed only slightly better than the time-invariant controller. Therefore, the time-invariant controller is a very good candidate for simultaneous attitude control and vibration suppression to reduce on-board computational requirements.

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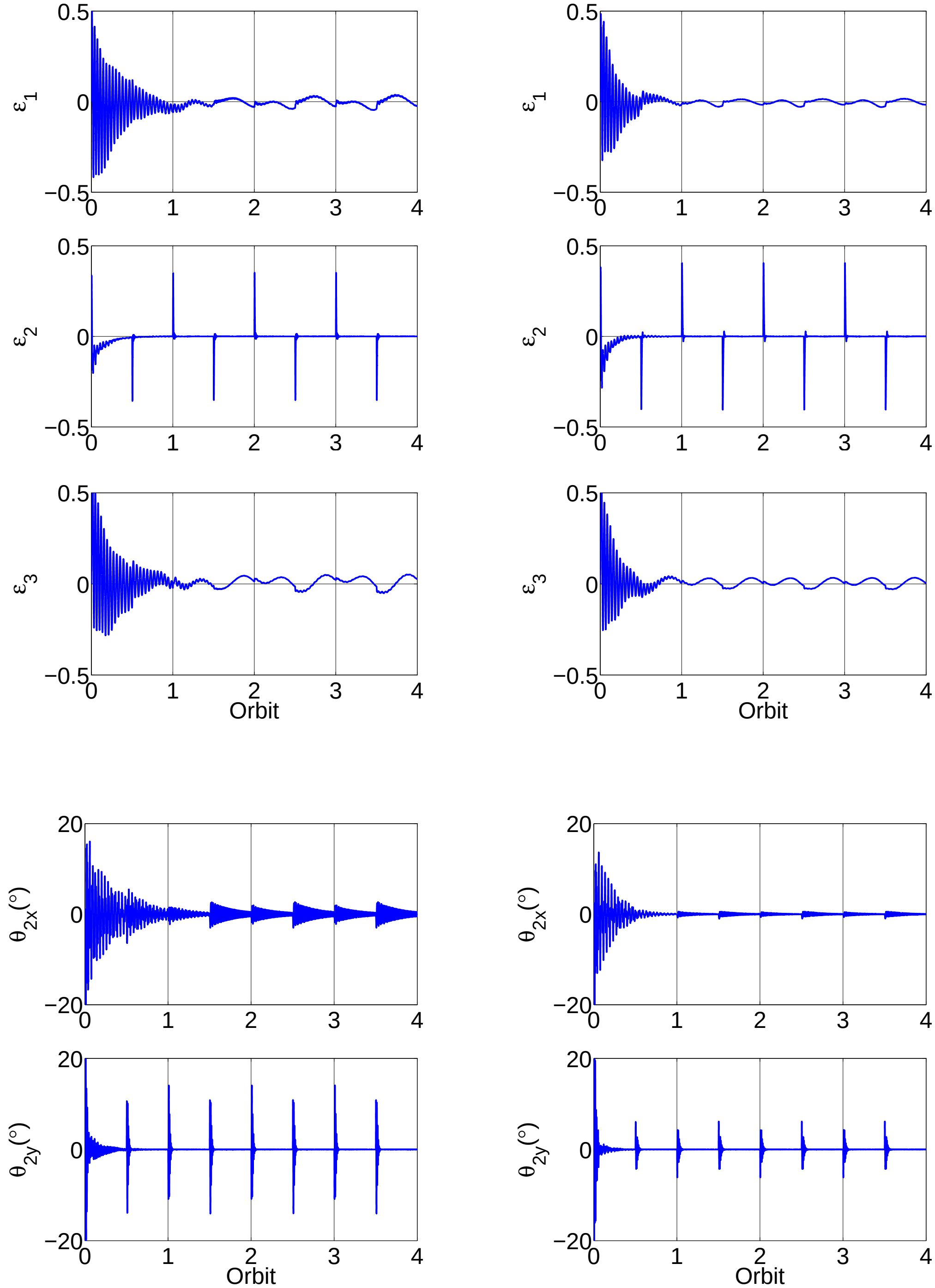

Fig. 2 Comparison of θ and $\tilde{\theta}^{2b}$ for the TIR (left) and TIVS (right) controllers.