Adaptive Spacecraft Attitude Tracking Control with Actuator Saturation

Anton. H. J. de Ruiter *

Carleton University

4925 Colonel By Drive, Ottawa, ON K1S 5B6, Canada

aderuite@mae.carleton.ca

Introduction

Practical spacecraft attitude control systems must operate in the presence of disturbances, modeling errors and actuator limitations. These issues have been the subject of much research interest. Adaptive control where the unknown system parameters are estimated adaptively, is one of the proposed approaches for dealing with modeling uncertainty (see for example [1, 2, 4, 3]). Both [1] and [2] deal with the attitude tracking problem, but do not treat disturbances or actuator saturation. Reference [3] also deals with the tracking problem, includes actuator saturation, but not disturbances. Reference [4] includes bounded disturbances, but does not treat actuator saturation, and only deals with the attitude regulator problem.

Recently new control laws have been obtained that treat both disturbances and actuator limitations simultaneously [5, 6, 7, 8]. References [5, 6] deal with the attitude regulation problem only. References [7, 8] treat the attitude tracking problem and both present globally convergent control laws given bounds on the spacecraft inertia matrix and the disturbances. The advantage of these approaches is that the form of the disturbance need not be known, only the bound. On the other hand, these approaches have no ability to learn the system model, which could be a useful feature if the attitude motion is to be optimized.

This paper shows that when an adaptive attitude control law based on the form given in [2] is appropriately designed, any linearly parameterizable disturbances can be accommodated, the closed-loop system is stable, with asymptotic tracking in the presence of actuator saturation. The unknown system parameters are learned adaptively.

*Anton de Ruiter is an Assistant Professor at Carleton University

Mathematical Preliminaries

In this paper, the vector norm used is the 2-norm. The matrix norm used is the induced 2-norm. The identity matrix is denoted by $\mathbf{1}$. The acronym b.u.c. will denote a bounded uniformly continuous function of time.

The following results are needed.

Lemma 1 [7].

Consider the filtered error given by $\mathbf{r}(t) \stackrel{\Delta}{=} \boldsymbol{\omega}(t) + \lambda \mathbf{q}(t)$, where

$$\begin{aligned} \dot{\mathbf{q}} &= \tfrac{1}{2}(\mathbf{q}^{\times} + q_4 \mathbf{1})\boldsymbol{\omega}, \\ \dot{q}_4 &= -\tfrac{1}{2}\mathbf{q}^T \boldsymbol{\omega}. \end{aligned} \tag{1}$$

and $\lambda > 0$. Then, $\mathbf{r}(t) \to \mathbf{0}$ as $t \to \infty$ if and only if $\boldsymbol{\omega}(t) \to \mathbf{0}$ and $\mathbf{q}(t) \to \mathbf{0}$ as $t \to \infty$.

Proposition 1 [9]

Let the function $\mathbf{f}(\mathbf{x}, t) : R^n \times R \to R^m$ be uniformly continuous in $\mathbf{x}$, uniformly in t, and uniformly continuous in t, uniformly in $\mathbf{x}$. Then, given any uniformly continuous function $\mathbf{y}(t) : R \to R^n$, the function $\mathbf{g}(t) \stackrel{\Delta}{=} \mathbf{f}(\mathbf{y}(t), t)$ is uniformly continuous in t.

Spacecraft Attitude Dynamics

In body coordinates, the attitude dynamics of a spacecraft controlled using reaction wheels are [10, pp.157, 237]

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^{\times}\left(\mathbf{I}\boldsymbol{\omega} + \mathbf{h}_w\right) + \frac{3\mu}{\|\mathbf{R}\|^5}\left(\mathbf{C}\mathbf{R}\right)^{\times}\mathbf{I}\left(\mathbf{C}\mathbf{R}\right) - \dot{\mathbf{h}}_w + \boldsymbol{\tau}_{des} + \boldsymbol{\tau}_d, \tag{2}$$

where $\mathbf{I}$ is the spacecraft inertia matrix, $\boldsymbol{\omega}$ is the inertial angular velocity, $\mathbf{h}_w$ is the total stored angular momentum in the wheels, $\mathbf{C}$ is the rotation matrix representing the spacecraft inertial attitude, μ is the gravitational parameter, $\mathbf{R}$ is the inertial spacecraft position, $\boldsymbol{\tau}_{des}$ is a wheel desaturation torque (provided by magnetorquers, or thrusters), and $\boldsymbol{\tau}_d$ is an external disturbance torque. The term $\dot{\mathbf{h}}_w$ is the control input.

Assumption 1

$\mathbf{R}$ is continuous, and $\|\mathbf{R}\|$, $\|\mathbf{R}\|^{-1}$ are bounded. Note that this is satisfied for any orbiting spacecraft.

Assumption 2

It is assumed that the disturbance torque can be parameterized linearly as

$$\boldsymbol{\tau}_d = \mathbf{W}_d(\mathbf{C}, t)\boldsymbol{\theta}_d, \tag{3}$$

for some vector $\boldsymbol{\theta}_d \in R^d$, and some bounded mapping $\mathbf{W}_d(\mathbf{C}, t) : R^{3\times 3} \times R \rightarrow R^{3\times d}$, continuous in $\mathbf{C}$ uniformly in t and uniformly continuous in t uniformly in $\mathbf{C}$.

Formulation of the Control Law

The desired inertial attitude rotation matrix is denoted by $\mathbf{C}_d(t)$. The desired angular velocity $\boldsymbol{\omega}_d(t)$ satisfies $\dot{\mathbf{C}}_d(t) = -\boldsymbol{\omega}_d^{\times}(t)\mathbf{C}_d(t)$ (see [10, p. 23]). It is assumed that $\mathbf{C}_d(t)$, $\boldsymbol{\omega}_d(t)$ and $\dot{\boldsymbol{\omega}}_d(t)$ are continuous and bounded. The attitude error is defined as

$$\delta\mathbf{C}(t) \stackrel{\Delta}{=} \mathbf{C}(t)\mathbf{C}_d^T(t). \tag{4}$$

With a quaternion parameterization $(\mathbf{q}, q_4)$ of the attitude error $\delta\mathbf{C}$, the error kinematics are given by [10, p. 31]

$$\begin{aligned} \dot{\mathbf{q}} &= \tfrac{1}{2}(\mathbf{q}^{\times} + q_4\mathbf{1})\delta\boldsymbol{\omega}, \\ \dot{q}_4 &= -\tfrac{1}{2}\mathbf{q}^T\delta\boldsymbol{\omega}. \end{aligned} \tag{5}$$

where $\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \delta\mathbf{C}\boldsymbol{\omega}_d$. The spacecraft inertia $\mathbf{I}$ and the disturbance parameter vector $\boldsymbol{\theta}_d$ are assumed unknown. Let

$$\mathbf{I}\dot{\bar{\boldsymbol{\omega}}}_d + \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{I}\boldsymbol{\omega} - \frac{3\mu}{\|\mathbf{R}\|^5}(\mathbf{C}\mathbf{R})^{\times}\mathbf{I}(\mathbf{C}\mathbf{R}) = \mathbf{W}_I\left(\dot{\bar{\boldsymbol{\omega}}}_d, \bar{\boldsymbol{\omega}}_d, \boldsymbol{\omega}, \mathbf{C}\mathbf{R}\right)\boldsymbol{\theta}_I, \tag{6}$$

where $\bar{\boldsymbol{\omega}}_d \stackrel{\Delta}{=} \delta\mathbf{C}\boldsymbol{\omega}_d - \lambda\mathbf{q}$ with $\lambda > 0$, $\mathbf{W}_I\left(\dot{\bar{\boldsymbol{\omega}}}_d, \bar{\boldsymbol{\omega}}_d, \boldsymbol{\omega}, \mathbf{C}\mathbf{R}\right)$ is a regressor matrix and $\boldsymbol{\theta}_I = vec\{\mathbf{I}\}$. Define the overall unknown parameter as $\boldsymbol{\theta} = \left[\boldsymbol{\theta}_I^T, \boldsymbol{\theta}_d^T\right]^T$, with corresponding regressor

$$\mathbf{W}\left(\dot{\bar{\boldsymbol{\omega}}}_d, \bar{\boldsymbol{\omega}}_d, \boldsymbol{\omega}, \mathbf{C}, \mathbf{R}, t\right) = \begin{bmatrix} -\mathbf{W}_I & \mathbf{W}_d \end{bmatrix}. \tag{7}$$

The control and adaptation laws are chosen to be

$$\dot{\mathbf{h}}_w = \mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w + \boldsymbol{\tau}_{des} + \mathbf{u}, \tag{8}$$

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\mathbf{W}^T\tilde{\boldsymbol{\omega}}, \tag{9}$$

where $\mathbf{u}$ is a feedback term, $\hat{\boldsymbol{\theta}}$ is an estimate of $\boldsymbol{\theta}$ and $\boldsymbol{\Gamma} > \mathbf{0}$ is some constant positive definite matrix. Defining the adaptation error to be $\tilde{\boldsymbol{\theta}} \stackrel{\Delta}{=} \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$, and substituting (8) and (3) into (2) leads to the filtered error dynamics

$$\mathbf{I}\dot{\tilde{\boldsymbol{\omega}}} = -\tilde{\boldsymbol{\omega}}^{\times}(\mathbf{I}\boldsymbol{\omega} + \mathbf{h}_w) + \mathbf{W}\left(\dot{\bar{\boldsymbol{\omega}}}_d, \bar{\boldsymbol{\omega}}_d, \boldsymbol{\omega}, \mathbf{C}, \mathbf{R}, t\right)\tilde{\boldsymbol{\theta}} - \mathbf{u}, \tag{10}$$

$$\dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}\mathbf{W}^T\tilde{\boldsymbol{\omega}}, \tag{11}$$

where the filtered error is $\tilde{\boldsymbol{\omega}} \triangleq \boldsymbol{\omega} - \bar{\boldsymbol{\omega}}_d = \delta\boldsymbol{\omega} + \lambda \mathbf{q}$.

Assumption 3

The class of feedback control laws considered in this paper is

$$\mathbf{u} = \mathbf{g}(\tilde{\boldsymbol{\omega}}, t), \tag{12}$$

where $\mathbf{g}(\tilde{\boldsymbol{\omega}}, t)$ is continuous in $\tilde{\boldsymbol{\omega}}$ uniformly in t, uniformly continuous in t uniformly in $\tilde{\boldsymbol{\omega}}$, and for any $d > 0$, $\tilde{\boldsymbol{\omega}} \in \mathcal{B}_d$ implies $\exists \bar{g} > 0$ such that $\|\mathbf{g}(\tilde{\boldsymbol{\omega}}, t)\| \leq \bar{g}, \forall t > 0$, where $\mathcal{B}_d \triangleq \{\mathbf{x} \in R^3 : \|\mathbf{x}\| \leq d\}$.

Assumption 4

The desaturation torque $\boldsymbol{\tau}_{des}$ maintains the wheel momentum $\mathbf{h}_w$ within operational bounds and is uniformly continuous and bounded.

The result in Lemma 1 leads to a large class of asymptotically convergent controllers.

Theorem 1

Let the feedback control law given in Assumption 3 be such that $\forall \tilde{\boldsymbol{\omega}} \neq \mathbf{0}$, $\exists \epsilon(||\tilde{\boldsymbol{\omega}}||) > 0$ such that $\inf_{t \in [0,\infty)} \tilde{\boldsymbol{\omega}}^T \mathbf{g}(\tilde{\boldsymbol{\omega}}, t) \geq \epsilon$. Then, under assumptions 1 to 4, the closed-loop system with (8) and (9) is stable, and

$$\delta\boldsymbol{\omega}(t) \to \mathbf{0}, \;\; \mathbf{q}(t) \to \mathbf{0},$$

as $t \to \infty$.

Proof Consider the Lyapunov-like function

$$V(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\theta}}) \triangleq \frac{1}{2}\tilde{\boldsymbol{\omega}}^T \mathbf{I} \tilde{\boldsymbol{\omega}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{\theta}}.$$

Using (10) and (11), its derivative is

$$\dot{V} = -\tilde{\boldsymbol{\omega}}^T \mathbf{g}(\tilde{\boldsymbol{\omega}}, t) \leq 0, \tag{13}$$

such that

$$V(t) \leq V(0), \;\; \forall t \geq 0, \tag{14}$$

showing that both $\tilde{\boldsymbol{\omega}}(t)$ and $\tilde{\boldsymbol{\theta}}(t)$ are bounded and the system is stable. From this, it can be shown that $\delta\boldsymbol{\omega}$, $\boldsymbol{\omega}(t)$ and $\mathbf{g}(\tilde{\boldsymbol{\omega}}, t)$ are bounded. Differentiating $\bar{\boldsymbol{\omega}}_d$ gives

$$\dot{\bar{\boldsymbol{\omega}}}_d = -\delta\boldsymbol{\omega}^\times \delta\mathbf{C}\boldsymbol{\omega}_d + \delta\mathbf{C}\dot{\boldsymbol{\omega}}_d - \frac{1}{2}\lambda\left(\mathbf{q}^\times + q_4 \mathbf{1}\right)\delta\boldsymbol{\omega}, \tag{15}$$

so $\dot{\bar{\boldsymbol{\omega}}}_d(t)$ is bounded. Boundedness of $\tilde{\boldsymbol{\theta}}$ implies boundedness of $\tilde{\mathbf{I}}$ and $\tilde{\boldsymbol{\theta}}_d$. Therefore, from (6) and Assumption 1,

$\mathbf{W}_I\tilde{\boldsymbol{\theta}}_I$ is bounded. $\mathbf{W}_d\tilde{\boldsymbol{\theta}}_d$ is bounded by Assumption 2. From (10) and Assumption 4, $\dot{\tilde{\boldsymbol{\omega}}}(t)$ is bounded. Therefore, $\tilde{\boldsymbol{\omega}}(t)$ is b.u.c. Since $\tilde{\boldsymbol{\omega}}(t)$ is bounded, $\tilde{\boldsymbol{\omega}}(t) \in D$, $\forall t \geq 0$ for some compact set D. By Assumption 3, on the set D, $\mathbf{g}(\tilde{\boldsymbol{\omega}}(t), t)$ is a uniformly continuous function in $\tilde{\boldsymbol{\omega}}$ uniformly in t, and uniformly continuous in t uniformly in $\tilde{\boldsymbol{\omega}}$. By Proposition 1, $\mathbf{g}(\tilde{\boldsymbol{\omega}}(t), t)$ is b.u.c. and therefore, so is the product $\tilde{\boldsymbol{\omega}}^T(t)\mathbf{g}(\tilde{\boldsymbol{\omega}}(t), t)$. From (14), the integral $\int_0^\infty -\tilde{\boldsymbol{\omega}}^T\mathbf{g}(\tilde{\boldsymbol{\omega}}, t)dt$ exists and is finite. Therefore, by Barbalat's Lemma [11, p. 192], it can be concluded that $\tilde{\boldsymbol{\omega}}^T\mathbf{g}(\tilde{\boldsymbol{\omega}}, t) \to \mathbf{0}$ as $t \to \infty$. Since $\forall \tilde{\boldsymbol{\omega}} \neq \mathbf{0}$, $\exists \epsilon(||\tilde{\boldsymbol{\omega}}||) > 0$ such that $\inf_{t\in[0,\infty)} \tilde{\boldsymbol{\omega}}^T\mathbf{g}(\tilde{\boldsymbol{\omega}}, t) \geq \epsilon$, it must be that $\tilde{\boldsymbol{\omega}} \to \mathbf{0}$ as $t \to \infty$. Finally, Lemma 1 shows $\delta\boldsymbol{\omega}(t) \to \mathbf{0}$, $\mathbf{q}(t) \to \mathbf{0}$ as $t \to \infty$. □

Actuator Saturation

Assumption 5

It is assumed that the available control is limited by

$$-\mathbf{u}_{max} \leq \dot{\mathbf{h}}_w \leq \mathbf{u}_{max}, \tag{16}$$

where $\mathbf{u}_{max} > \mathbf{0}$ and the inequality in (16) is taken componentwise.

Consider now the feedback control law

$$\mathbf{u} = \mathbf{K}\tilde{\boldsymbol{\omega}}, \tag{17}$$

where $\mathbf{K} = diag\{k_1, k_2, k_3\}$ with $k_i > 0,\ \ i = 1, 2, 3$. Under Assumption 5, the control law (8) is now implemented as

$$\dot{\mathbf{h}}_w = sat\left(\mathbf{W}\hat{\boldsymbol{\theta}}(t) - \bar{\boldsymbol{\omega}}_d^\times\mathbf{h}_w + \boldsymbol{\tau}_{des} + \mathbf{K}\tilde{\boldsymbol{\omega}}, -\mathbf{u}_{max}, \mathbf{u}_{max}\right), \tag{18}$$

where the saturation function is defined componentwise as

$$sat\left(\mathbf{x}_i, \mathbf{x}_{min,i}, \mathbf{x}_{max,i}\right) \triangleq \begin{cases} \mathbf{x}_{max,i}, & \text{if } \mathbf{x}_i > \mathbf{x}_{max,i} \\ \mathbf{x}_i & \text{if } \mathbf{x}_{min,i} \leq \mathbf{x}_i \leq \mathbf{x}_{max,i} \\ \mathbf{x}_{min,i} & \text{if } \mathbf{x}_i < \mathbf{x}_{min,i} \end{cases} \tag{19}$$

Assumption 6

$\boldsymbol{\omega}_d$ and $\dot{\boldsymbol{\omega}}_d$ are uniformly continuous and bounded, and satisfy $-\mathbf{u}_{max} + \boldsymbol{\delta} \leq \mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^\times\mathbf{h}_w + \boldsymbol{\tau}_{des} \leq \mathbf{u}_{max} - \boldsymbol{\delta}$ for $\boldsymbol{\delta} > \mathbf{0}$ where $\boldsymbol{\delta} = \delta\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ for some $\delta > 0$.

Under Assumption 6, the control law (18) becomes

$$\mathbf{u}_f = \mathbf{W}\hat{\boldsymbol{\theta}}(t) - \bar{\boldsymbol{\omega}}_d^\times\mathbf{h}_w + \boldsymbol{\tau}_{des} + sat\left(\mathbf{K}\tilde{\boldsymbol{\omega}}, \bar{\mathbf{u}}_{min}(t), \bar{\mathbf{u}}_{max}(t)\right), \tag{20}$$

where

$$\bar{\mathbf{u}}_{min}(t) \triangleq -\mathbf{u}_{max} - \left(\mathbf{W}\hat{\boldsymbol{\theta}}(t) - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w + \boldsymbol{\tau}_{des}\right), \tag{21}$$

and

$$\bar{\mathbf{u}}_{max}(t) \triangleq \mathbf{u}_{max} - \left(\mathbf{W}\hat{\boldsymbol{\theta}}(t) - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w + \boldsymbol{\tau}_{des}\right), \tag{22}$$

with $2\mathbf{u}_{min} < \bar{\mathbf{u}}_{min}(t) < -\boldsymbol{\delta}$ and $2\mathbf{u}_{max} > \bar{\mathbf{u}}_{max}(t) > \boldsymbol{\delta}$. The result including actuator saturation limitations is now presented.

Theorem 2

Let assumptions 1, 2, 4, 5 and 6 be satisfied. Then, the closed-loop system with (18) and (9) is stable, with

$$\delta\boldsymbol{\omega}(t) \to \mathbf{0},\ \mathbf{q}(t) \to \mathbf{0},$$

as $t \to \infty$.

Proof

It is clear that $\mathbf{g}(\tilde{\boldsymbol{\omega}}, t) = sat\left(\mathbf{K}\tilde{\boldsymbol{\omega}}, \bar{\mathbf{u}}_{min}(t), \bar{\mathbf{u}}_{max}(t)\right)$ is bounded. Letting $\underline{k}$ and $\bar{k}$ be the minimum and maximum diagonal entries in $\mathbf{K}$ respectively, and choosing $\epsilon(||\tilde{\boldsymbol{\omega}}||) = \underline{k}||\tilde{\boldsymbol{\omega}}||^2$ when $||\tilde{\boldsymbol{\omega}}|| \le \frac{\delta}{k}$, and $\epsilon(||\tilde{\boldsymbol{\omega}}||) = \underline{k}\delta^2$ when $||\tilde{\boldsymbol{\omega}}|| > \frac{\delta}{k}$, it can readily be shown that $\inf_{t\in[0,\infty)} \tilde{\boldsymbol{\omega}}^T\mathbf{g}(\tilde{\boldsymbol{\omega}}, t) \ge \epsilon$. The only condition in Theorem 1 that remains to be verified is Assumption 3.

As in Theorem 1, it can be shown that $\tilde{\boldsymbol{\omega}}$, $\tilde{\boldsymbol{\theta}}$, $\delta\boldsymbol{\omega}$, $\boldsymbol{\omega}$, $\dot{\tilde{\boldsymbol{\omega}}}$ are bounded. Therefore, $\tilde{\boldsymbol{\omega}}$ is b.u.c. From (5), $\dot{\mathbf{q}}$, $\dot{q}_4$ are bounded. From $\dot{\mathbf{C}} = -\boldsymbol{\omega}^{\times}\mathbf{C}$ (see [10, p. 23]), $\dot{\mathbf{C}}$ is bounded. Hence, $\mathbf{C}(t)$, $\mathbf{q}(t)$, $q_4(t)$ and consequently $\delta\mathbf{C}(t)$ are b.u.c. From Proposition 1 and Assumption 2, $\mathbf{W}_d$ is b.u.c. It can now be shown that, $\delta\boldsymbol{\omega}(t)$, $\boldsymbol{\omega}$, $\bar{\boldsymbol{\omega}}_d(t)$ and $\dot{\bar{\boldsymbol{\omega}}}_d(t)$ are all b.u.c. From (6), $\mathbf{W}_I$ is polynomial in the elements of $\dot{\bar{\boldsymbol{\omega}}}_d$, $\bar{\boldsymbol{\omega}}_d$, $\boldsymbol{\omega}$ and $\mathbf{CR}$. From Assumption 1 then, $\mathbf{W}_I(t)$ is b.u.c. From (16), $\dot{\mathbf{h}}_w$ is bounded, and from Assumption 4, $\mathbf{h}_w(t)$ is b.u.c. Since $\boldsymbol{\theta}$ is constant and $\tilde{\boldsymbol{\theta}}$ is bounded, $\hat{\boldsymbol{\theta}}$ is bounded also. From (9), $\dot{\hat{\boldsymbol{\theta}}}$ is bounded. Hence, $\hat{\boldsymbol{\theta}}(t)$ is b.u.c. From the above results and Assumption 4, $\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w + \boldsymbol{\tau}_{des}$ is b.u.c. Therefore from (21) and (22), $\bar{\mathbf{u}}_{min}(t)$ and $\bar{\mathbf{u}}_{min}(t)$ are uniformly continuous.

Each component of $\mathbf{g}(\tilde{\boldsymbol{\omega}}, t)$ has the form $h(x, t) = sat\left(kx, a(t), b(t)\right)$ where $a(t) < 0$ and $b(t) > 0$ are uniformly continuous. Fixing t, it can be seen that $h(x, t)$ satisfies

$$|h(x_2, t) - h(x_1, t)| \le k|x_2 - x_1|$$

Therefore, with t fixed, $h(x, t)$ is uniformly continuous in x, independently of t. Fixing x, and choosing any $\epsilon > 0$, by uniform continuity of $a(t)$ and $b(t)$, $\exists\delta_a > 0,\ \ \delta_b > 0$ such that

$$|t_2 - t_1| < \delta_a \Rightarrow |a(t_2) - a(t_1)| < \epsilon,$$

$$|t_2 - t_1| < \delta_b \Rightarrow |b(t_2) - b(t_1)| < \epsilon,$$

Let $\delta \triangleq \min(\delta_a, \delta_b)$. Then,

$$|t_2 - t_1| < \delta \Rightarrow |a(t_2) - a(t_1)| < \epsilon,\ |b(t_2) - b(t_1)| < \epsilon.$$

There are now several cases to consider. Consider $x > 0$.

Case 1 $b(t_1) < kx$ and $b(t_2) < kx$.

In this case, $|h(x, t_2) - h(x, t_1)| = |b(t_2) - b(t_1)| < \epsilon$.

Case 2 $b(t_1) < kx \leq b(t_2)$.

In this case, $|h(x, t_2) - h(x, t_1)| = |kx - b(t_1)| \leq |b(t_2) - b(t_1)| < \epsilon$.

Case 3 $b(t_2) < kx \leq b(t_1)$.

Swapping t_1 and t_2, this is the same as case 2.

Case 4 $b(t_1) \geq kx$ and $b(t_2) \geq kx$.

In this case, $|h(x, t_2) - h(x, t_1)| = 0$.

The case $x < 0$ is dealt with similarly. The case $x = 0$ is trivial, since $h(0, t) = 0$. Therefore, given any $\epsilon > 0$, $\exists \delta > 0$ independently of x, such that

$$|t_2 - t_1| < \delta \Rightarrow |h(x, t_2) - h(x, t_1)| < \epsilon.$$

Therefore, Assumption 3 is satisfied, and Theorem 1 yields the result. □

A condition is now presented under which Assumption 6 is satisfied. Consider the ball

$$\mathcal{B}_{\underline{u}} = \{\mathbf{u} \in R^3 : \|\mathbf{u}\| \leq \underline{u} - \delta, \underline{u} = \min_{i=1,2,3}(\mathbf{u}_{max,i})\}.$$

This is the largest ball such that

$$\mathcal{B}_{\underline{u}} \subset \{\mathbf{u} \in R^3 : -\mathbf{u}_{max} + \boldsymbol{\delta} < \mathbf{u} < \mathbf{u}_{max} - \boldsymbol{\delta}\}.$$

Assumption 6 is satisfied if $\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w + \boldsymbol{\tau}_{des} \in \mathcal{B}_{\underline{u}}$. Since the desaturation torque $\boldsymbol{\tau}_{des}$ can be used to reduce $\|\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w + \boldsymbol{\tau}_{des}\|$, its contribution is ignored. That is, in the following analysis, $\boldsymbol{\tau}_{des} \equiv \mathbf{0}$

Bounds on the unknown parameters are assumed to be available.

$$0 < I_{min}\mathbf{1} \leq \mathbf{I} \leq I_{max}\mathbf{1}, \tag{23}$$

$$\|\boldsymbol{\theta}_d\| \leq \bar{\theta}_d. \tag{24}$$

Since $\mathbf{I} \in R^{3\times 3}$ is symmetric, only six components are unknown, i.e. $\boldsymbol{\theta}_I \in R^6$. Defining $\boldsymbol{\theta}_{I,1} = \begin{bmatrix} \{\boldsymbol{\theta}_I\}_1 & \{\boldsymbol{\theta}_I\}_2 & \{\boldsymbol{\theta}_I\}_3 \end{bmatrix}^T$ and $\boldsymbol{\theta}_{I,2} = \begin{bmatrix} \{\boldsymbol{\theta}_I\}_4 & \{\boldsymbol{\theta}_I\}_5 & \{\boldsymbol{\theta}_I\}_6 \end{bmatrix}^T$, assume that $\boldsymbol{\Gamma} = blockdiag\{\gamma_1 \mathbf{1}_{3\times 3}, \gamma_2 \mathbf{1}_{3\times 3}, \gamma_1 \mathbf{1}_{d\times d}\},\ \gamma_i > 0,\ i = 1, 2, 3$. The Lyapunov-like function in Theorem 1 is now

$$V(t) = \frac{1}{2}\tilde{\boldsymbol{\omega}}^T(t)\mathbf{I}\boldsymbol{\omega}(t) + \frac{1}{2\gamma_1}\tilde{\boldsymbol{\theta}}_{I,1}^T(t)\tilde{\boldsymbol{\theta}}_{I,1}(t) + \frac{1}{2\gamma_2}\tilde{\boldsymbol{\theta}}_{I,2}^T(t)\tilde{\boldsymbol{\theta}}_{I,2}(t) + \frac{1}{2\gamma_3}\tilde{\boldsymbol{\theta}}_d^T(t)\tilde{\boldsymbol{\theta}}_d(t) \tag{25}$$

Given bounds on the initial estimation error, $\|\tilde{\boldsymbol{\theta}}_{I,i}(0)\| \le \bar{\tilde{\theta}}_{i,0},\ i = 1, 2$ and $\|\tilde{\boldsymbol{\theta}}_d(0)\| \le \bar{\tilde{\theta}}_{d,0}$, one has $V(0) \le \bar{V}$, where

$$\bar{V} = \frac{1}{2}I_{max}\tilde{\boldsymbol{\omega}}^T(0)\tilde{\boldsymbol{\omega}}(0) + \frac{1}{2\gamma_1}\bar{\tilde{\theta}}_{1,0}^2 + \frac{1}{2\gamma_2}\bar{\tilde{\theta}}_{2,0}^2 + \frac{1}{2\gamma_3}\bar{\tilde{\theta}}_{d,0}^2. \tag{26}$$

From (14), (25) and (26), one obtains

$$I_{min}\|\tilde{\boldsymbol{\omega}}\|^2 + \frac{1}{\gamma_1}\|\tilde{\boldsymbol{\theta}}_{I,1}\|^2 + \frac{1}{\gamma_2}\|\tilde{\boldsymbol{\theta}}_{I,2}\|^2 + \frac{1}{\gamma_3}\|\tilde{\boldsymbol{\theta}}_d\|^2 \le 2\bar{V}. \tag{27}$$

Given that $\|\mathbf{q}\| \le 1$, the following can be obtained

$$\|\bar{\boldsymbol{\omega}}_d\| \le \|\boldsymbol{\omega}_d\| + \lambda, \tag{28}$$

$$\|\boldsymbol{\omega}\| \le \|\boldsymbol{\omega}_d\| + \lambda + \|\tilde{\boldsymbol{\omega}}\|, \tag{29}$$

$$\|\delta\boldsymbol{\omega}\| \le \lambda + \|\tilde{\boldsymbol{\omega}}\|. \tag{30}$$

It can be shown that $\|\mathbf{q}^\times + q_4\mathbf{1}\| = 1$. Therefore, (30) and (15) lead to

$$\|\dot{\bar{\boldsymbol{\omega}}}_d\| \le \|\dot{\boldsymbol{\omega}}_d\| + \left(\|\boldsymbol{\omega}_d\| + \frac{\lambda}{2}\right)(\|\tilde{\boldsymbol{\omega}}\| + \lambda). \tag{31}$$

From $\tilde{\boldsymbol{\theta}} \triangleq \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$, (23) and (24), bounds on the estimates

$$\|\hat{\mathbf{I}}(t)\| \le I_{max} + \|\tilde{\mathbf{I}}(t)\|, \tag{32}$$

and

$$\|\hat{\boldsymbol{\theta}}_d(t)\| \le \bar{\theta}_d + \|\tilde{\boldsymbol{\theta}}_d(t)\|, \tag{33}$$

are obtained. Given

$$\tilde{\mathbf{I}} = \begin{bmatrix} \tilde{I}_{xx} & \tilde{I}_{xy} & \tilde{I}_{xz} \\ \tilde{I}_{xy} & \tilde{I}_{yy} & \tilde{I}_{yz} \\ \tilde{I}_{zz} & \tilde{I}_{yz} & \tilde{I}_{zz} \end{bmatrix},$$

the corresponding vectors may be defined as

$$\tilde{\boldsymbol{\theta}}_{I,1} = \begin{bmatrix} \tilde{I}_{xx} & \tilde{I}_{yy} & \tilde{I}_{zz} \end{bmatrix}^T, \tag{34}$$

and

$$\tilde{\boldsymbol{\theta}}_{I,2} = \begin{bmatrix} \tilde{I}_{xy} & \tilde{I}_{xz} & \tilde{I}_{yz} \end{bmatrix}^T. \tag{35}$$

By definition of norms,

$$\|\tilde{\mathbf{I}}\| = \sqrt{\lambda_{max}(\tilde{\mathbf{I}}\tilde{\mathbf{I}}^T)} \leq \sqrt{trace(\tilde{\mathbf{I}}\tilde{\mathbf{I}}^T)} = \sqrt{\|\tilde{\boldsymbol{\theta}}_{I,1}\|^2 + 2\|\tilde{\boldsymbol{\theta}}_{I,2}\|^2}. \tag{36}$$

Constants $\bar{w}, \bar{h}_w \geq 0$ are now assumed known such that

$$\|\mathbf{W}_d(\mathbf{C}, t)\| \leq \bar{w}, \tag{37}$$

$$\|\mathbf{h}_w(t)\| \leq \bar{h}_w. \tag{38}$$

Using the above bounds with (6) and (7), it can be shown that if $\tilde{\boldsymbol{\omega}}^T \mathbf{g}\left(\tilde{\boldsymbol{\omega}}, t\right) \geq 0$ on the interval $t \in [0, t^*]$ for some $t^* > 0$, then

$$\|\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w\| \leq \bar{W}_\theta, \tag{39}$$

on that interval, where

$$\begin{aligned} \bar{W}_\theta \triangleq\; & \left[\|\dot{\boldsymbol{\omega}}_d\| + \left(2\|\boldsymbol{\omega}_d\| + \tfrac{3\lambda}{2}\right)\lambda + \left(\|\boldsymbol{\omega}_d\| + \lambda\right)\|\boldsymbol{\omega}_d\| + \tfrac{3\mu}{\|\mathbf{R}\|^3}\right] I_{max} + \bar{w}\bar{\theta}_d \\ & + \left(\|\boldsymbol{\omega}_d\| + \lambda\right)\bar{h}_w + \left[\|\dot{\boldsymbol{\omega}}_d\| + \left(2\|\boldsymbol{\omega}_d\| + \tfrac{3\lambda}{2}\right)\lambda + \left(\|\boldsymbol{\omega}_d\| + \lambda\right)\|\boldsymbol{\omega}_d\| + \tfrac{3\mu}{\|\mathbf{R}\|^3}\right] \\ & \times\sqrt{\|\tilde{\boldsymbol{\theta}}_{I,1}\|^2 + 2\|\tilde{\boldsymbol{\theta}}_{I,2}\|^2} \\ & + \left(2\|\boldsymbol{\omega}_d\| + \tfrac{3\lambda}{2}\right)\left(I_{max} + \sqrt{\|\tilde{\boldsymbol{\theta}}_{I,1}\|^2 + 2\|\tilde{\boldsymbol{\theta}}_{I,2}\|^2}\right)\|\tilde{\boldsymbol{\omega}}\| + \bar{w}\|\tilde{\boldsymbol{\theta}}_d\|. \end{aligned} \tag{40}$$

Given the bound in (39), if

$$\max_{\|\tilde{\boldsymbol{\omega}}\|, \|\tilde{\boldsymbol{\theta}}_{I,1}\|, \|\tilde{\boldsymbol{\theta}}_{I,2}\|, \|\tilde{\boldsymbol{\theta}}_2\|} \bar{W}_\theta < \underline{u} - \delta, \tag{41}$$

subject to the constraint (27), it can be concluded that $\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^{\times}\mathbf{h}_w \in \mathcal{B}_{\underline{u}}$ on the interval $t \in [0, t^*]$ for some $t^* > 0$. A contradiction argument as in [9] can be used to extend this to the infinite interval $t \in [0, \infty)$, showing Assumption

6 is satisfied. Indeed, suppose it were not the case, then by continuity of the solution, there must exist a time $\bar{t} > 0$ and $0 < \delta_2 < \delta$, such that $\|\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^\times \mathbf{h}_w\| = \underline{u} - \delta_2$, and $\|\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^\times \mathbf{h}_w\| \leq \underline{u} - \delta_2$ on the interval $t \in [0, \bar{t}]$, such that Assumption 6 is satisfied on this interval with δ_2. From the proof of Theorem 2, it is seen that $\tilde{\boldsymbol{\omega}}^T \mathbf{g}(\tilde{\boldsymbol{\omega}}, t) \geq 0$ on this interval. Therefore, from the above argument, it must be the case that $\|\mathbf{W}\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\omega}}_d^\times \mathbf{h}_w\| \leq \underline{u} - \delta < \underline{u} - \delta_2$ on this interval, which is a contradiction. Hence, Assumption 6 must be satisfied on the infinite interval $t \in [0, \infty)$ with δ.

Conclusion

This note extends a nonlinear adaptive controller for spacecraft attitude tracking that has previously been presented in the literature. Any linearly parameterizable disturbance can be accommodated, and the unknown system parameters can be learned online. The control law consists of both a feedback and a feedforward component. It is shown that when the feedforward component is restricted to lie within the actuator capabilities, the closed-loop system is asymptotically convergent in the presence of actuator saturation (which is due to saturation of the feedback component of the control law). A condition on the desired attitude trajectory is obtained which guarantees that the feedforward component of the control law does not saturate, which in turn guarantees asymptotic attitude tracking.

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