Information Sciences xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Distributed attitude synchronization control for a group of flexible spacecraft using only attitude measurements

An-Min Zou*, Anton H. J. de Ruiter, Krishna Dev Kumar

Department of Aerospace Engineering, Ryerson University, Toronto, ON M5B 2K3, Canada

ARTICLE INFO

Article history:
Received 1 February 2015
Revised 13 November 2015
Accepted 24 January 2016
Available online xxx

ABSTRACT

This paper considers the problem of attitude synchronization control of a group of flexible spacecraft under an undirected communication graph and in the absence of measurements of both modal variables and spacecraft angular velocities. To solve this problem, a nonlinear observer is introduced to estimate the unmeasurable modal variables and spacecraft angular velocities. Then, the backstepping technique is used to design the control law. The stability of the overall closed-loop system is guaranteed by the Lyapunov approach together with Barbalat's Lemma. The performance of the control scheme derived here is illustrated through numerical simulations.

1. Introduction

Recently, the problem of attitude cooperative control for a group of spacecraft has attracted significant research attention. This is due to the fact that multiple cooperative spacecraft is an applicable technology for many space missions such as monitoring of the Earth and its surrounding atmosphere, geodesy, deep space imaging and exploration, and in-orbit servicing and maintenance of spacecraft.

For a group of rigid spacecraft, a class of decentralized coordination tracking control laws was developed in [21]. Ren [14] proposed control laws for a team of spacecraft through local information exchange. With consideration of external disturbances and time delays, a decentralized variable structure controller for attitude coordination control of multiple spacecraft was presented in [9]. Based on a state-dependent Riccati equation technique, a decentralized attitude coordination control algorithm for satellite formation flying was proposed in [4]. With consideration of the control input saturation, a neural network-based distributed attitude coordination control scheme was proposed for a group of rigid spacecraft when the common reference attitude was available only to a portion of the group members [26]. Using a Lagrangian approach and nonlinear contraction analysis, the problem of cooperative tracking control for a group of spacecraft was studied in [5]. In [2], the leader-follower attitude consensus problem for a multiple rigid spacecraft system was discussed. Using the finite-time control technique, the finite-time attitude cooperative control problem has been investigated in [8,13,23–25]. In these works, full state measurements (i.e., both attitude and angular velocity) are required for use in the control laws.

Based on a bi-directional ring topology, a passivity based formation control law for multi-spacecraft attitude alignment was proposed in [12]. Later, the work of [12] was extended in [15] to the case of a general undirected connected communication topology. In [1], a velocity-free attitude tracking and synchronization control scheme was developed for a group of

* Corresponding author. Tel.: +1 4166090241.
E-mail addresses: amzou2012@gmail.com, amzou@126.com (A.-M. Zou), aderuiter@ryerson.ca (A. H. J. de Ruiter), kdkumar@ryerson.ca (K. Dev Kumar).

http://dx.doi.org/10.1016/j.ins.2016.01.048
0020-0255/© 2016 Published by Elsevier Inc.

spacecraft. For the case when the time-varying reference attitude was available only to a subset of the group members, a velocity-free attitude coordination control scheme was designed for a group of rigid spacecraft [27].

It is noteworthy that the aforementioned attitude coordination laws are designed for rigid spacecraft. Due to the effect of flexible dynamics, the problem of attitude coordination control for multiple flexible spacecraft is much more challenging. In [22], a nonfragile output tracking control scheme was presented for flexible hypersonic air-breathing vehicles, and in [20], an output feedback controller was presented for a class of TCS fuzzy stochastic systems. Furthermore, a joint model of an autonomous distributed system was used for a production process [17], and a distributed fuzzy filter was designed for a class of sensor networks [19]. However, it is not straightforward to extend the results in [17,19,20,22] to the problem of attitude coordination control of a group of flexible spacecraft.

More recently, using the unit quaternion as the attitude representation, a distributed attitude synchronization control law was proposed in [7] for a group of flexible spacecraft. However, it is only shown that the vector part of the quaternion of each spacecraft in the group can achieve synchronization, and it is not clear whether true attitude synchronization can be reached. Furthermore, angular velocity measurements are required in the control law in [7]. However, in practical applications, due to either cost limitations or implementation considerations, angular velocity measurements may not be available. Therefore, it is necessary to develop a new velocity-free control scheme for the problem of attitude synchronization control of a groupof flexible spacecraft.

In this paper, the attitude synchronization control for a group of flexible spacecraft under an undirected communication graph is addressed. The knowledge of modal variables and spacecraft angular velocity is assumed to be unavailable, and the attitude of each spacecraft in the group is represented by modified Rodrigues parameters (MRPs). It should be emphasized that the effects of the flexible dynamics, affecting the rigid motion, and the absence of measurements of modal variables and of the spacecraft angular velocity, makes the controller design more challenging. To solve this problem, a semi-global nonlinear observer is first developed for each spacecraft system to estimate the attitude state and modal variables of itself. Note that the term "semi-global" is referred to the attitude system described by MRPs. Next, a distributed attitude synchronization control law is proposed based on the backstepping technique. Finally, a rigorous stability analysis for the overall closed-loop is provided by using the Lyapunov approach together with Barbalat's Lemma.

2. Background and preliminaries

2.1. Notations

The notation $\|\cdot\|$ refers to the Euclidean norm of a vector or the induced norm of a matrix. I_n represents the $n \times n$ identity matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of a matrix, respectively. For $x_i \in R^{n_i}, i = 1, 2, \ldots, n$, $\mathrm{col}(x_1, x_2, \ldots, x_n) = [x_1^T, x_2^T, \ldots, x_n^T]^T$. For $x \in R^3$, $x^\times \in R^{3\times 3}$ denotes a 3×3 skew-symmetric matrix defined by

$$x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{1}$$

2.2. Attitude kinematics and dynamics of flexible spacecraft

In this paper, we consider a group of n flexible spacecraft. Using MRPs [18] as the attitude representation, the attitude kinematics for the ith spacecraft is described by

$$\dot{q}_i = T_i(q_i)\omega_i, \tag{2}$$

where $q_i(t) \in R^3$ represents the MRPs [18] describing the spacecraft attitude with respect to an inertial frame, defined by

$$q_i(t) = \varrho_i(t)\tan\left(\frac{\phi_i(t)}{4}\right), \quad \phi_i \in [0, 2\pi)\,\mathrm{rad} \tag{3}$$

with ϱ_i and ϕ_i denoting the Euler eigenaxis and eigenangle, respectively, $\omega_i \in R^3$ is the angular velocity of the ith spacecraft in a body-fixed frame, and the Jacobian matrix $T_i(q_i) \in R^{3\times 3}$ for the MRPs is given by [18]

$$T_i(q_i) = \frac{1}{2}\left[\frac{1 - q_i^T q_i}{2} I_3 + q_i^\times + q_i q_i^T\right]. \tag{4}$$

The attitude description using the MRPs has an advantage of being valid for eigenaxis rotations up to 360°.

For the ith flexible spacecraft, the attitude dynamics is [6]

$$\begin{aligned} J_i\dot{\omega}_i + \delta_i^T\ddot{\eta}_i &= -\omega_i^\times\left(J_i\omega_i + \delta_i^T\dot{\eta}_i\right) + \tau_i, \\ \ddot{\eta}_i + C_i\dot{\eta}_i + K_i\eta_i &= -\delta_i\dot{\omega}_i, \end{aligned} \tag{5}$$

where $J_i \in R^{3\times 3}$ is the inertia matrix, $\tau_i \in R^3$ is the control torque, δ_i is the coupling matrix between the rigid body and the flexible attachments, η_i is the modal displacement, $C_i = \mathrm{diag}\{2\zeta_{i,k}\varpi_{i,nk}, k = 1, 2, \ldots, N_i\}$ and $K_i = \mathrm{diag}\{\varpi_{i,nk}^2, k = 1, 2, \ldots, N_i\}$

Please cite this article as: A.-M. Zou et al., Distributed attitude synchronization control for a group of flexible spacecraft using only attitude measurements, Information Sciences (2016), http://dx.doi.org/10.1016/j.ins.2016.01.048

are the damping and stiffness matrices, N_i is the number of flexible modes considered, $\varpi_{i,\,nk}$ is the natural frequency, and $\zeta_{i,\,k}$ is the corresponding damping.

To facilitate the controller development, like [6], we introduce the following variable

$$\psi_i = \dot{\eta}_i + \delta_i\omega_i \tag{6}$$

representing the total angular velocity expressed in modal variables. Then, the attitude dynamics (5) can be reexpressed as:

$$\begin{aligned} \dot{\eta}_i &= \psi_i - \delta_i\omega_i, \;\; \dot{\psi}_i = -C_i\psi_i - K_i\eta_i + C_i\delta_i\omega_i \\ \bar{J}_i\dot{\omega}_i &= -\omega_i^{\times}(\bar{J}_i\omega_i + \delta_i^T\psi_i) + \delta_i^T(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) + \tau_i, \end{aligned} \tag{7}$$

where $\bar{J}_i = J_i - \delta_i^T\delta_i$.

By appropriate procedures and definitions, the system in (2) and (7) can be transformed as:

$$\begin{aligned} \dot{q}_i &= \nu_i, \quad \dot{\nu}_i = f_i(q_i, \nu_i, \eta_i, \psi_i) + g_i(q_i)\tau_i, \\ \dot{\eta}_i &= \psi_i - \delta_i P_i\nu_i, \quad \dot{\psi}_i = -C_i\psi_i - K_i\eta_i + C_i\delta_i P_i\nu_i, \end{aligned} \tag{8}$$

where $\nu_i = \dot{q}_i, P_i = T_i^{-1}, g_i = T_i\bar{J}_i^{-1}$, and $f_i(q_i, \nu_i, \eta_i, \psi_i) = -T_i\dot{P}_i\nu_i - T_i\bar{J}_i^{-1}(P_i\nu_i)^{\times}(\bar{J}_iP_i\nu_i + \delta_i^T\psi_i) + T_i\bar{J}_i^{-1}\delta_i^T(C_i\psi_i + K_i\eta_i - C_i\delta_iP_i\nu_i)$.

The main objective of the present work is to design a control law for $\tau_i (i = 1, 2, \ldots, n)$ such that the attitude states of all spacecraft in the group can reach synchronization even in the absence of measurements of both spacecraft angular velocity and modal variables.

2.3. Communication topology

Using the graph theory, we model the topology of the information flow among spacecraft by a weighted undirected connected graph $G = (\Upsilon, E, A)$, where $\Upsilon = \{r_1, r_2, \ldots, r_n\}$ is the set of nodes, $E \subseteq \Upsilon \times \Upsilon$ is the set of edges, and $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of graph G with nonnegative elements. Node $r_i (i = 1, 2, \ldots, n)$ represents the ith spacecraft, and an edge in G is denoted by an unordered pair (r_i, r_j). $(r_i, r_j) \in E$ if and only if there is an information exchange between the ith spacecraft and the jth spacecraft. Since the graph is undirected, $(r_i, r_j) \in E \Leftrightarrow (r_j, r_i) \in E$. The adjacency element a_{ij} denotes the communication quality between the ith spacecraft and the jth spacecraft, i.e., $(\upsilon_i, \upsilon_j) \in E \Leftrightarrow a_{ij} > 0$. It is assumed that $a_{ij} = a_{ji}$ and $a_{ii} = 0$; that is, the weighted adjacency matrix A is a symmetric matrix.

The degree matrix of the weighted graph G is defined as $D = \text{diag}\{d_1, d_2, \ldots, d_n\}$, where $d_i = \sum_{j=1}^{n} a_{ij} (i = 1, 2, \ldots, n)$. The Laplacian matrix L of the weighted graph G is defined as $L = D - A$, which is a symmetric matrix. For any two nodes r_i and r_j, if there exists a path between them, then G is called a connected graph. In this paper, it is assumed that the graph G is connected.

3. Distributed attitude synchronization control for a group of flexible spacecraft

In this section, we focus on designing a distributed attitude synchronization control law for a group of n flexible spacecraft. We assume that measurements of both spacecraft angular velocities and modal variables are unavailable for use in designing the control law. To solve this problem, a nonlinear observer is introduced to estimate the angular velocity and modal variables, and then the backstepping technique is applied to design a distributed control law such that the attitudes of all spacecraft can reach synchronization even in the absence of measurements of both spacecraft angular velocities and modal variables.

3.1. Nonlinear observer

Since measurements of both spacecraft angular velocities and modal variables are unavailable, we design a nonlinear observer for each individual spacecraft, defined as follows:

$$\begin{aligned} \dot{\hat{q}}_i &= \hat{\nu}_i + \theta\theta_1\tilde{q}_i, \\ \dot{\hat{\nu}}_i &= f_i(q_i, \hat{\nu}_i, \hat{\eta}_i, \hat{\psi}_i) + g_i\tau_i + \theta^2\theta_2\tilde{q}_i, \end{aligned} \tag{9}$$

where θ, θ_1 and θ_2 are positive constants, $\hat{q}_i$ with $\hat{q}_i(0) = q_i(0)$ and $\hat{\nu}_i$ with $\hat{\nu}_i(0) = 0$ are estimates of q_i and ν_i, respectively, $\tilde{q}_i = q_i - \hat{q}_i$, and $\hat{\psi}_i$ and $\hat{\eta}_i$ are estimates of ψ_i and η_i generated by

$$\dot{\hat{\eta}}_i = \hat{\psi}_i - \delta_iP_i\hat{\nu}_i, \quad \dot{\hat{\psi}}_i = -C_i\hat{\psi}_i - K_i\hat{\eta}_i + C_i\delta_iP_i\hat{\nu}_i. \tag{10}$$

The main idea behind the introduction of the observer given in (9) and (10) is to provide an asymptotic estimation of the angular velocity ν_i and the modal variables ψ_i and η_i. Through an appropriate choice of the control torque τ_i by using the estimated variables $\hat{\nu}_i$, $\hat{\psi}_i$ and $\hat{\eta}_i$, one can remove the requirements of measurements of both spacecraft angular velocities and modal variables. Furthermore, it is noted that if the effect of flexible dynamics is not considered (i.e., the spacecraft is rigid), which implies that $\psi_i \equiv 0$ and $\eta_i \equiv 0$, the observer in (9) can be considered as a Luenberger-style observer [3].

Consequently, the observer and controller proposed in this paper can be applied for a group of rigid spacecraft. A limitation for the proposed observer is that the inertia matrix or its nominal part must be known. It would be interesting to allow the inertia matrix to be unknown. Such a problem needs further investigation, and adaptive control and/or intelligent control approaches may be required to relax the above-mentioned limitation.

Now, we have the following Lemma.

Lemma 1. *For any given constant* $\Delta > 0$, *if* $q_i, \hat{q}_i, \nu_i, \hat{\nu}_i, \eta_i, \hat{\eta}_i, \psi_i$, *and* $\hat{\psi}_i$ *lie within the following compact set:*

$$\Omega_i = \{X_i \| \|q_i\| \le \Delta, \|\hat{q}_i\| \le \Delta, \|Y_i\| \le \Delta, \|\hat{Y}_i\| \le \Delta\}, \tag{11}$$

where $X_i = \mathrm{col}(q_i, \hat{q}_i, Y_i, \hat{Y}_i)$, $Y_i = \mathrm{col}(\nu_i, \eta_i, \psi_i)$, *and* $\hat{Y}_i = \mathrm{col}(\hat{\nu}_i, \hat{\eta}_i, \hat{\psi}_i)$, *then there exists a positive constant* f_{Mi} *such that* $\|f_i(q_i, Y_i) - f_i(q_i, \hat{Y}_i)\| \le f_{Mi}\|Y_i - \hat{Y}_i\|$.

Proof. Note that the set Ω_i is convex and compact, and we can obtain the Jacobian matrix $\partial f_i/\partial Y_i$ is bounded on the set Ω_i. Let f_{Mi} be the bound for $\|\partial f_i/\partial Y_i\|$, and define $h(s) = (1-s)Y_i + s\hat{Y}_i$. Since Ω_i is convex, we have $\|h(s)\| \le \Delta$ for $s \in [0, 1]$. Let $z \in R^3$ be

$$z = \frac{f_i(q_i, Y_i) - f_i(q_i, \hat{Y}_i)}{\|f_i(q_i, Y_i) - f_i(q_i, \hat{Y}_i)\|} \tag{12}$$

then we have $\|z\| = 1$ and

$$z^T(f_i(q_i, Y_i) - f_i(q_i, \hat{Y}_i)) = \|f_i(q_i, Y_i) - f_i(q_i, \hat{Y}_i)\|. \tag{13}$$

Define $G(s) = z^T f_i(q_i, h(s))$. Since $G(s)$ is a real-valued function and is continuously differentiable on [0, 1], it follows from the mean value theorem that there exists $s_1 \in (0, 1)$ such that $G(1) - G(0) = G'(s_1)$. Therefore, we can obtain

$$G(1) - G(0) = z^T(f_i(q_i, \hat{Y}_i) - f_i(q_i, Y_i)) = z^T \frac{\partial f_i}{\partial Y_i}(h(s1))(\hat{Y}_i - Y_i) \tag{14}$$

which in turn implies that

$$\|f_i(q_i, Y_i) - f_i(q_i, \hat{Y}_i)\| \le \|z\| \left\| \frac{\partial f_i}{\partial Y_i}(h(s1)) \right\| \|\hat{Y}_i - Y_i\| \le f_{M_i}\|Y_i - \hat{Y}_i\|. \tag{15}$$

□

The asymptotic stability of the observer defined by (9) and (10) is given in the following theorem.

Theorem 1. *Consider the observer described by (9) and (10). For any given constant* $\Delta > 0$, *if* $q_i, \hat{q}_i, \nu_i, \hat{\nu}_i, \eta_i, \hat{\eta}_i, \psi_i$, *and* $\hat{\psi}_i$ *lie within the compact set* Ω_i, *then there exists a sufficiently large observer parameter* θ *such that system given in (9) and (10) admits semi-global asymptotic observer.*

Proof. The observer estimated errors $\tilde{q}_i$, $\tilde{\nu}_i = \nu_i - \hat{\nu}_i$, $\tilde{\eta}_i = \eta_i - \hat{\eta}_i$, and $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$ are governed by the following differential equations:

$$\begin{aligned}
\dot{\tilde{q}}_i &= -\theta\theta_1\tilde{q}_i + \tilde{\nu}_i \\
\dot{\tilde{\nu}}_i &= -\theta^2\theta_2\tilde{q}_i + f_i(q_i, \nu_i, \eta_i, \psi_i) - f_i(q_i, \hat{\nu}_i, \hat{\eta}_i, \hat{\psi}_i) \\
\dot{\tilde{\eta}}_i &= \tilde{\psi}_i - \delta_i P_i \tilde{\nu}_i, \quad \dot{\tilde{\psi}}_i = -C_i\tilde{\psi}_i - K_i\tilde{\eta}_i + C_i\delta_i P_i\tilde{\nu}_i.
\end{aligned} \tag{16}$$

To facilitate the stability analysis, we consider the following coordinate transformations: $\zeta_{1i} = \tilde{q}_i/\theta$, $\zeta_{2i} = \tilde{\nu}_i/\theta^2$, $\varphi_{1i} = \tilde{\eta}_i/\theta^2$, and $\varphi_{2i} = \tilde{\psi}_i/\theta^2$. Then, the observer error dynamics expressed in the new coordinate is given as follows:

$$\begin{aligned}
\dot{\zeta}_{1i} &= -\theta\theta_1\zeta_{1i} + \theta\zeta_{2i} \\
\dot{\zeta}_{2i} &= -\theta\theta_2\zeta_{1i} + \frac{f_i(q_i, \nu_i, \eta_i, \psi_i) - f_i(q_i, \hat{\nu}_i, \hat{\eta}_i, \hat{\psi}_i)}{\theta^2} \\
\dot{\varphi}_{1i} &= \varphi_{2i} - \delta_i P_i \zeta_{2i} \\
\dot{\varphi}_{2i} &= -C_i\varphi_{2i} - K_i\varphi_{1i} + C_i\delta_i P_i\zeta_{2i}.
\end{aligned} \tag{17}$$

Let $M_{1i} \in R^{6 \times 6}$ and $M_{2i} \in R^{2N_i \times (2N_i)}$ be

$$M_{1i} = \begin{bmatrix} -\theta_1 I_3 & I_3 \\ -\theta_2 I_3 & 0 \end{bmatrix}, \quad M_{2i} = \begin{bmatrix} 0 & I_{N_i} \\ -K_i & -C_i \end{bmatrix}. \tag{18}$$

Because M_{1i} and M_{2i} are Hurwitz matrices, there exist positive definite matrices $N_{1i} = N_{1i}^T$ and $N_{2i} = N_{2i}^T$ such that $M_{1i}^T N_{1i} + N_{1i}M_{1i} = -I_6$ and $M_{2i}^T N_{2i} + N_{2i}M_{2i} = -I_{2N_i}$.

Consider the following Lyapunov function:

$$V_{oi} = \zeta_i^T N_{1i}\zeta_i + \varphi_i^T N_{2i}\varphi_i, \tag{19}$$

where $\zeta_i = \mathrm{col}(\zeta_{1i}, \zeta_{2i})$ and $\varphi_i = \mathrm{col}(\varphi_{1i}, \varphi_{2i})$.

The time derivative of V_{oi} along with system (17) is

$$\dot{V}_{oi} = -\theta\zeta_i^T\zeta_i - \varphi_i^T\varphi_i + 2\varphi_i^T N_{2i}\begin{bmatrix}-\delta_i P_i \zeta_{2i} \\ C_i\delta_i P_i\zeta_{2i}\end{bmatrix} + 2\zeta_i^T N_{1i}\begin{bmatrix}0 \\ \frac{f_i - \hat{f}_i}{\theta^2}\end{bmatrix}, \tag{20}$$

where $f_i = f_i(q_i, \upsilon_i, \eta_i, \psi_i)$ and $\hat{f}_i = f_i(q_i, \hat{\upsilon}_i, \hat{\eta}_i, \hat{\psi}_i)$.

Note that if $q_i, \hat{q}_i, \upsilon_i, \hat{\upsilon}_i, \eta_i, \hat{\eta}_i, \psi_i$, and $\hat{\psi}_i$ lie within the compact set Ω_i, then following the similar procedure of the proof of Lemma 1, it can be obtained that there exist some positive constants c_{1i} and c_{2i} such that $\|f_i - \hat{f}_i\| \le c_{1i}\|\tilde{\upsilon}_i\| + c_{2i}\left\|(\tilde{\eta}_i^T, \tilde{\psi}_i^T)^T\right\|$, and thus it follows that

$$\left\|\frac{f_i - \hat{f}_i}{\theta^2}\right\| \le c_{1i}\|\zeta_{2i}\| + c_{2i}\|\varphi_i\|. \tag{21}$$

Using the well-known inequality $a^T b \le \|a\|^2/(2\epsilon) + \epsilon\|b\|^2/2$, where $a \in R^n$, $b \in R^n$, and $\epsilon > 0$ is some constant, it follows that

$$\begin{aligned}\left|2\varphi_i^T N_{2i}\begin{bmatrix}-\delta_i P_i\zeta_{2i} \\ C_i\delta_i P_i\zeta_{2i}\end{bmatrix}\right| &\le 2\|\varphi_i\|\|N_{2i}\|(\|\mathrm{diag}\{-\delta_i P_i, C_i\delta_i P_i\}\|\|\zeta_{2i}\|) \\ &\le 2\|\varphi_i\|\|N_{2i}\|(\|\mathrm{diag}\{-\delta_i P_i, C_i\delta_i P_i\}\|\|\zeta_i\|) \le \frac{1}{4}\|\varphi_i\|^2 + c_{3i}\|\zeta_i\|^2\end{aligned} \tag{22}$$

$$\begin{aligned}\left|2\zeta_i^T N_{1i}\begin{bmatrix}0 \\ \frac{f_i - \hat{f}_i}{\theta^2}\end{bmatrix}\right| &\le 2\|\varphi_i\|\|N_{1i}\|(c_{1i}\|\zeta_{2i}\| + c_{2i}\|\varphi_i\|) \\ &\le 2\|\varphi_i\|\|N_{1i}\|(c_{1i}\|\zeta_i\| + c_{2i}\|\varphi_i\|) \le \frac{1}{4}\|\varphi_i\|^2 + c_{4i}\|\zeta_i\|^2,\end{aligned} \tag{23}$$

where c_{3i} and c_{4i} are some positive constants. Then, the time derivative of V_{oi} is upper bounded by

$$\begin{aligned}\dot{V}_{oi} &\le -(\theta - c_{3i} - c_{4i})\zeta_i^T\zeta_i - \frac{1}{2}\varphi_i^T\varphi_i \le -(\theta - c_{3M} - c_{4M})\zeta_i^T\zeta_i - \frac{1}{2}\varphi_i^T\varphi_i \\ &= -\bar{\theta}\zeta_i^T\zeta_i - \frac{1}{2}\varphi_i^T\varphi_i,\end{aligned} \tag{24}$$

where $c_{3M} = \max\{c_{3i}\}$, $c_{4M} = \max\{c_{4i}\}$, and $\bar{\theta} = \theta - c_{3M} - c_{4M}$. If the observer parameter θ is chosen sufficiently large such that $\theta > c_{3M} + c_{4M}$, then $\dot{V}_{oi} < 0$, which implies that the observer errors $\tilde{q}_i$, $\tilde{\upsilon}_i$, $\tilde{\eta}_i$, and $\tilde{\psi}_i$ semi-globally asymptotically converge to the origin. □

Remark 1. From (24), it can be obtained that

$$\dot{V}_{oi} \le -\bar{\theta}\zeta_i^T\zeta_i - \frac{1}{2}\varphi_i^T\varphi_i \le -\frac{\bar{\theta}}{\lambda_{\max}(N_{1i})}\zeta_i^T N_{1i}\zeta_i - \frac{1}{2\lambda_{\max}(N_{2i})}\varphi_i^T N_{2i}\varphi_i \le -m_i V_{oi}, \tag{25}$$

where $m_i = \min(\bar{\theta}/\lambda_{\max}(N_{1i}), 1/[2\lambda_{\max}(N_{2i})])$. Integrating both sides of (25), we obtain that $V_{oi}(t) \le V_{oi}(0)e^{-m_i t}$, $t \ge 0$, which implies that the observer errors $\tilde{q}_i$, $\tilde{\upsilon}_i$, $\tilde{\eta}_i$, and $\tilde{\psi}_i$ exponentially converge to zero as $t \to \infty$.

Next the effect of external disturbances and parameter perturbations on the performance of the proposed observer is analyzed. In this case, we assume that there exist bounded uncertainties acting on the spacecraft, i.e., the attitude dynamics is given by

$$J_i\dot{\omega}_i + \delta_i^T\ddot{\eta}_i = -\omega_i^{\times}(J_i\omega_i + \delta_i^T\dot{\eta}_i) + \tau_i + \vartheta_i \tag{26}$$

where $J_i \in R^{3\times 3}$ is the nominal part of the inertia matrix, and ϑ_i denotes the lumped uncertainties which include external disturbances and the uncertainties caused by parameter perturbations. Then, the dynamic equations for the observer errors $\tilde{q}_i$, $\tilde{\upsilon}_i$, $\tilde{\eta}_i$, and $\tilde{\psi}_i$ are given as follows:

$$\begin{aligned}\dot{\tilde{q}}_i &= -\theta\theta_1\tilde{q}_i + \tilde{\upsilon}_i \\ \dot{\tilde{\upsilon}}_i &= -\theta^2\theta_2\tilde{q}_i + f_i(q_i, \upsilon_i, \eta_i, \psi_i) - f_i(q_i, \hat{\upsilon}_i, \hat{\eta}_i, \hat{\psi}_i) + \bar{\vartheta}_i \\ \dot{\tilde{\eta}}_i &= \tilde{\psi}_i - \delta_i P_i\tilde{\upsilon}_i, \ \ \dot{\tilde{\psi}}_i = -C_i\tilde{\psi}_i - K_i\tilde{\eta}_i + C_i\delta_i P_i\tilde{\upsilon}_i\end{aligned} \tag{27}$$

where $\bar{\vartheta}_i = g_i\vartheta_i$. The result is stated as follows.

Corollary 1. *Consider the observer described by (9) and (10), and assume bounded uncertainties acting on the spacecraft. For any given constant $\Delta > 0$, if $q_i, \hat{q}_i, \upsilon_i, \hat{\upsilon}_i, \eta_i, \hat{\eta}_i, \psi_i$, and $\hat{\psi}_i$ lie within the compact set Ω_i, then there exists a sufficiently large observer parameter θ such that the observer errors $x_i = \mathrm{col}(\zeta_i, \varphi_i)$ asymptotically converge to a small bounded set Σ_i which will be defined later.*

Please cite this article as: A.-M. Zou et al., Distributed attitude synchronization control for a group of flexible spacecraft using only attitude measurements, Information Sciences (2016), http://dx.doi.org/10.1016/j.ins.2016.01.048

Proof. The analysis procedure is similar to that of the proof of Theorem 1, except for the presence of the uncertain term $\tilde{\vartheta}_i$ $(i = 1, 2, \ldots, n)$. Considering the Lyapunov function given in (19), and using (24), we have

$$\dot{V}_{oi} \le -\bar{\theta}\zeta_i^T\zeta_i - \frac{1}{2}\varphi_i^T\varphi_i + 2\zeta_i^T N_{1i}\begin{bmatrix} 0 \\ \frac{\tilde{\vartheta}_i}{\theta^2}\end{bmatrix} \le -(\bar{\theta}-1)\zeta_i^T\zeta_i - \frac{1}{2}\varphi_i^T\varphi_i + \frac{\vartheta_{Mi}}{\theta^4}$$
$$\le -\vartheta\,\|x_i\|^2 + \frac{\vartheta_{Mi}}{\theta^4}, \tag{28}$$

where the inequality $a^Tb \le \|a\|^2/(2\epsilon) + \epsilon\|b\|^2/2$ has been used; ϑ_{Mi} is some positive constant, and $\vartheta = \min(\bar{\theta}-1, 1/2)$. If the observer parameter θ is chosen sufficiently large such that $\bar{\theta} > 1$, then $\dot{V}_{oi} < 0$ as long as

$$\|x_i\| > \frac{\sqrt{\vartheta_{Mi}}}{\sqrt{\vartheta}\theta^2} \tag{29}$$

which implies that the observer errors x_i will asymptotically converge to the set

$$\Sigma_i = \left\{ x_i \,\middle|\, \|x_i\| \le \frac{\sqrt{\vartheta_{Mi}}}{\sqrt{\vartheta}\theta^2} \right\}. \tag{30}$$

□

Remark 2. From (30), it can be concluded that the size of the set Σ_i is determined by the observer parameter θ; that is, the smaller the observer errors x_i, the bigger the observer parameter θ is required.

3.2. Controller design

The backstepping technique [10] is applied to design the controller, and the procedure of the controller design is stated as follows.

Step 1: Design of virtual control law for $\hat{v}_i (i = 1, 2, \ldots, n)$.

For the following subsystem:

$$\dot{q}_i = v_i = \tilde{v}_i + \hat{v}_i$$
$$\dot{\hat{\eta}}_i = \hat{\psi}_i - \delta_i P_i \hat{v}_i,\ \ \dot{\hat{\psi}}_i = -C_i\hat{\psi}_i - K_i\hat{\eta}_i + C_i\delta_i P_i \hat{v}_i \tag{31}$$

the variable $\hat{v}_i$ is considered as a virtual input.

Consider the following Lyapunov function candidate:

$$V_1 = \sum_{i=1}^{n} U_i,\ U_i = \frac{q_i^Tq_i}{2} + \frac{1}{2}\hat{\psi}_i^T\hat{\psi}_i + \hat{\eta}_i^T K_i\hat{\eta}_i + \frac{1}{2}(\hat{\psi}_i + C_i\hat{\eta}_i)^T(\hat{\psi}_i + C_i\hat{\eta}_i). \tag{32}$$

The time derivative of U_i along with system (31) is

$$\dot{U}_i = -\hat{\eta}_i^T C_i K_i \hat{\eta}_i - \hat{\psi}_i^T C_i \hat{\psi}_i + (\hat{\psi}^T C_i - 2\hat{\eta}_i^T K_i)\delta_i P_i \hat{v}_i + q_i^T\tilde{v}_i + q_i^T\hat{v}_i$$
$$= -\hat{\eta}_i^T C_i K_i \hat{\eta}_i - \hat{\psi}_i^T C_i \hat{\psi}_i + \beta_i^T\hat{v}_i + \beta_i^T\tilde{v}_i - \bar{\beta}_i^T\tilde{v}_i, \tag{33}$$

where $\beta_i = q_i + P_i^T\delta_i^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i)$ and $\bar{\beta}_i = P_i^T\delta_i^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i)$.

Design a virtual control law for $\hat{v}_i$ as:

$$\hat{v}_{di} = -k_1\sum_{j=1}^{n} a_{ij}(\beta_i - \beta_j), \tag{34}$$

where $k_1 > 0$ is a constant.

The time derivative of V_1 is

$$\dot{V}_1 = \sum_{i=1}^{n}\left(-\hat{\eta}_i^T C_i K_i\hat{\eta}_i - \hat{\psi}_i^T C_i\hat{\psi}_i + \beta_i^T\hat{v}_i + \beta_i^T\tilde{v}_i - \bar{\beta}_i^T\tilde{v}_i\right)$$
$$= \sum_{i=1}^{n}\left(-\hat{\eta}_i^T C_i K_i\hat{\eta}_i - \hat{\psi}_i^T C_i\hat{\psi}_i + \beta_i^T\hat{v}_{ei} + \beta_i^T\tilde{v}_i - \bar{\beta}_i^T\tilde{v}_i\right) - k_1\sum_{i=1}^{n}\beta_i^T\sum_{j=1}^{n}a_{ij}(\beta_i - \beta_j), \tag{35}$$

where $\hat{v}_{ei} = \hat{v}_i - \hat{v}_{di}$. Noting that

$$-\sum_{i=1}^{n}\beta_i^T\sum_{j=1}^{n}a_{ij}(\beta_i - \beta_j) = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\beta_i - \beta_j)^T(\beta_i - \beta_j) \tag{36}$$

and using the following inequality

$$\bar{\beta}_i^T\tilde{v}_i = \tilde{v}_i^T\bar{\beta}_i = \tilde{v}_i^T P_i^T\delta_i^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i) \le \frac{1}{2}\hat{\eta}_i^T C_i K_i\hat{\eta}_i + \frac{1}{2}\hat{\psi}_i^T C_i\hat{\psi}_i + c_{5i}\|\tilde{v}_i\|^2, \tag{37}$$

where the inequality $a^Tb \le \|a\|^2/(2\epsilon) + \epsilon\|b\|^2/2$ has been applied, and c_{5i} is a positive constant, it follows from (35) that

$$\dot{V}_1 \le \sum_{i=1}^n\left(-\frac{1}{2}\hat{\eta}_i^T C_i K_i\hat{\eta}_i - \frac{1}{2}\hat{\psi}_i^T C_i\hat{\psi}_i + \beta_i^T\hat{v}_{ei} + \beta_i^T\tilde{v}_i + c_{5i}\|\tilde{v}_i\|^2\right) - \frac{k_1}{2}\sum_{i=1}^n\sum_{j=1}^n a_{ij}\|\beta_i - \beta_j\|^2. \tag{38}$$

Step 2: Design of the control torque $\tau_i (i = 1, 2, \ldots, n)$.

The governing dynamic equation for the error $\hat{v}_{ei}$ is

$$\dot{\hat{v}}_{ei} = \hat{f}_i + g_i\tau_i + \theta^2\theta_2\tilde{q}_i - \dot{\hat{v}}_{di}, \tag{39}$$

where $\dot{\hat{v}}_{di} = -k_1\sum_{j=1}^n a_{ij}(\alpha_i - \alpha_j)$ with $\alpha_i = v_i + P_i^T\delta_i^T(C_i\dot{\hat{\psi}}_i - 2K_i\dot{\hat{\eta}}_i) + \dot{P}_i^T\delta_i^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i)$. Note that $\dot{P}_i = -P_i\dot{T}_iP_i$.

The control torque τ_i is now designed as

$$\tau_i = g_i^{-1}\left(-k_2\hat{v}_{ei} - \hat{f}_i - \theta^2\theta_2\tilde{q}_i - k_1\sum_{j=1}^n a_{ij}(\hat{\alpha}_i - \hat{\alpha}_j)\right), \tag{40}$$

where $k_2 > 0$ is a constant, $\hat{\alpha}_i = \hat{v}_i + P_i^T\delta_i^T(C_i\dot{\hat{\psi}}_i - 2K_i\dot{\hat{\eta}}_i) - (P_i\hat{T}_iP_i)^T\delta_i^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i)$, and $\hat{T}_i = (-q_i^T\hat{v}_i + \hat{v}_i^\times + 2q_i\hat{v}_i^T)/2$.

Note that the measurements of both spacecraft angular velocity and modal variable are unnecessary to implement the control torque (40). Furthermore, the control law depends only on the local information rather than the global information, and hence, the control law is distributed. The main result of the present work is stated in the following theorem.

Theorem 2. *Consider a group of flexible spacecraft described by (2) and (5). If the undirected graph G is connected, the observer is given by (9) and (10), the control torque is defined by (40), and for any positive constant V_M, the initial conditions satisfy*

$$\bar{V} = V_1 + \sum_{i=1}^n V_{oi} + \sum_{i=1}^n\frac{1}{2}\hat{v}_{ei}^T\hat{v}_{ei} \le V_M, \tag{41}$$

where V_{oi} and V_1 are defined by (19) and (32), respectively, then there exist parameters θ, θ_1, θ_2, k_1 and k_2 such that the attitude synchronization can be achieved asymptotically.

Proof. Substituting the control input (40) into (39) results in

$$\dot{\hat{v}}_{ei} = -k_2\hat{v}_{ei} + k_1\sum_{j=1}^n a_{ij}(\tilde{\alpha}_i - \tilde{\alpha}_j), \tag{42}$$

where $\tilde{\alpha}_i = \alpha_i - \hat{\alpha}_i = \tilde{v}_i + (P_i(\dot{T}_i - \hat{T}_i)P_i)^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i)$. Note that $\dot{T}_i = (-q_i^Tv_i + v_i^\times + 2q_iv_i^T)/2$, and $\tilde{\alpha}_i$ can be reexpressed as

$$\tilde{\alpha}_i = \tilde{v}_i + \left(P_i(-q_i^T\tilde{v}_i + \tilde{v}_i^\times + 2q_i\tilde{v}_i^T)P_i\right)^T(C_i\hat{\psi}_i - 2K_i\hat{\eta}_i)/2. \tag{43}$$

Consider the following Lyapunov function candidate:

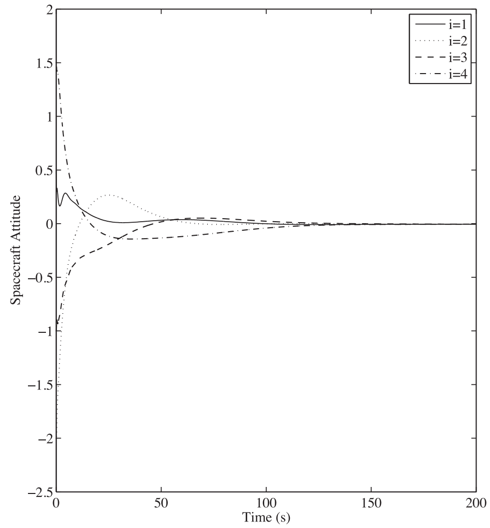
$$V = V_2 + K\sum_{i=1}^n V_{oi},\ V_2 = V_1 + \sum_{i=1}^n\frac{1}{2}\hat{v}_{ei}^T\hat{v}_{ei}, \tag{44}$$

where V_{oi} and V_1 are defined by (19) and (32), respectively, and $K > 0$ is a large constant.

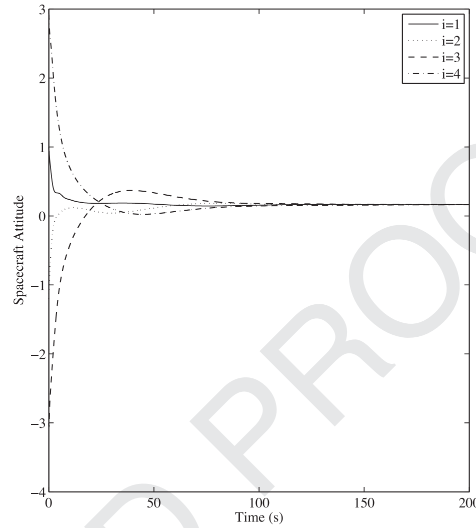
Noticing that the initial q_i, $\hat{\eta}_i$ and $\hat{\psi}_i$ satisfy the condition (41), and it follows from (43) that $\|\tilde{\alpha}_i\| \le \alpha_{Mi}\|\tilde{v}_i\|$, and consequently

$$\begin{aligned}\hat{v}_{ei}^T\left(k_1\sum_{j=1}^n a_{ij}(\tilde{\alpha}_i - \tilde{\alpha}_j)\right) &\le \|\hat{v}_{ei}\|\left(k_1a_M\sum_{j=1}^n(\alpha_{Mi}\|\tilde{v}_i\| + \alpha_{Mj}\|\tilde{v}_j\|)\right)\\ &\le \frac{k_2}{2}\|\hat{v}_{ei}\|^2 + c_{6i}\|\tilde{v}_i\|^2 + \sum_{j=1}^n\bar{c}_{6j}\|\tilde{v}_j\|^2,\end{aligned} \tag{45}$$

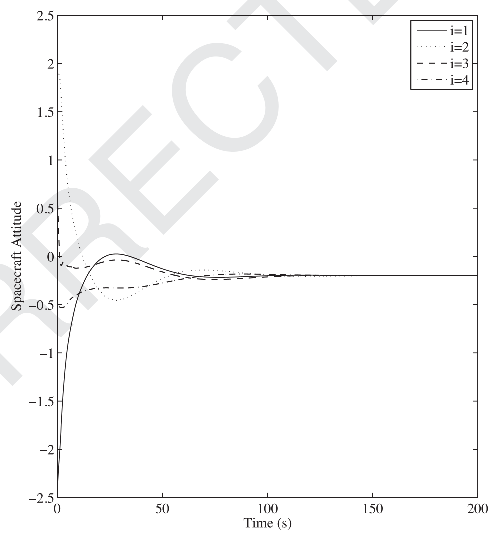
where α_{Mi}, c_{6i} and $\bar{c}_{6j}$ are some positive constants, $a_M = \max a_{ij}$, $i, j \in [1, 2, \ldots, n]$, and the inequality $ab \le a^2/(2\epsilon) + \epsilon b^2/2$ has been used. Using (38), (42) and (45), the time derivative of V_2 is



(a) $q_{i,1}$



(b) $q_{i,2}$

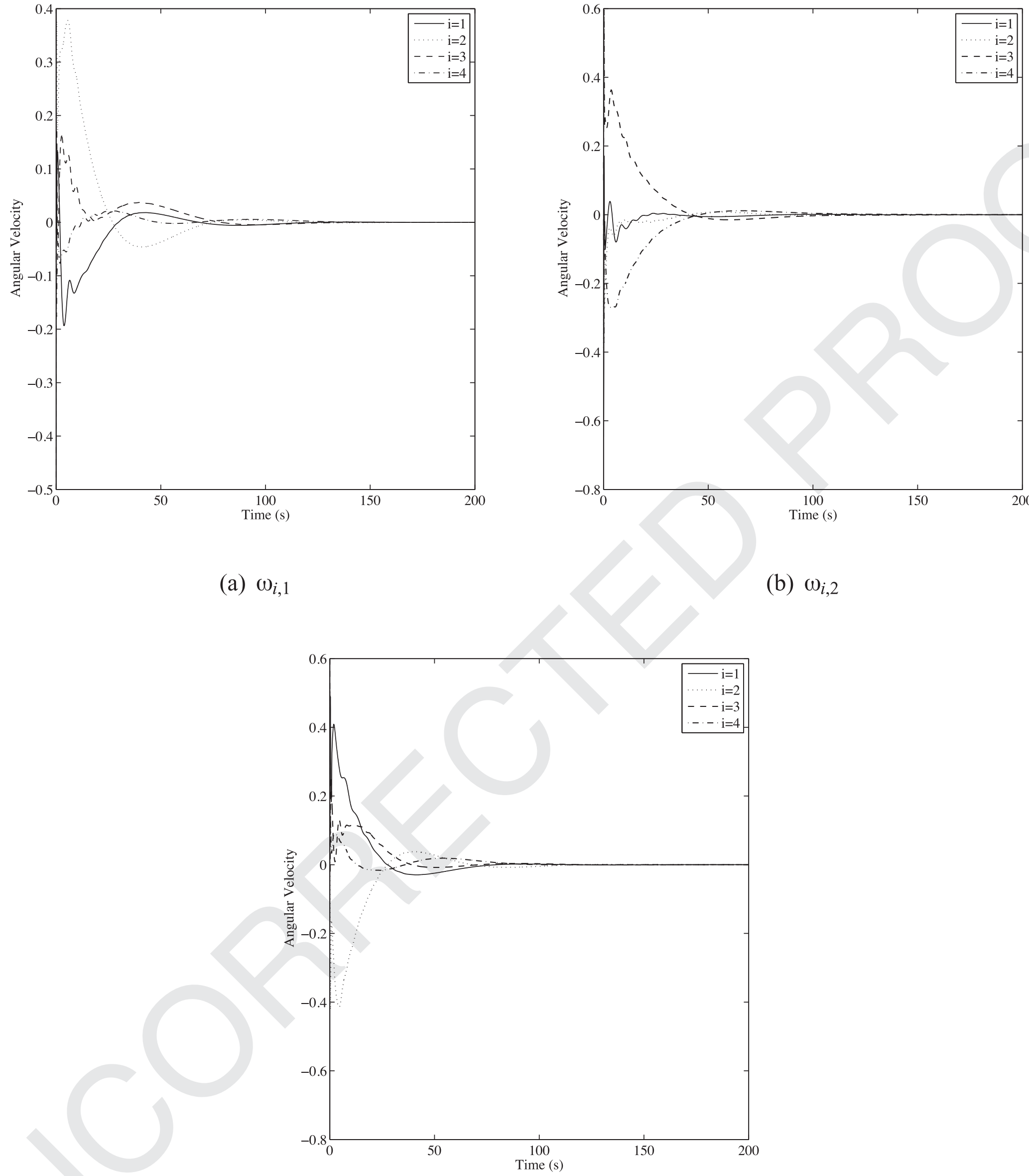


(c) $q_{i,3}$

Fig. 1. Spacecraft attitude.

$$\begin{aligned}\dot{V}_2 \leq & \sum_{i=1}^{n}\left(-\frac{1}{2}\hat{\eta}_i^T C_i K_i \hat{\eta}_i - \frac{1}{2}\hat{\psi}_i^T C_i \hat{\psi}_i + (c_{5i}+c_{6i})\|\tilde{v}_i\|^2 + \sum_{j=1}^{n}\bar{c}_{6j}\|\tilde{v}_j\|^2\right) \\ & - \frac{k_2}{2}\sum_{i=1}^{n}\|\hat{v}_{ei}\|^2 - \frac{k_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\|\beta_i-\beta_j\|^2 + \sum_{i=1}^{n}(d_{1i}\|\tilde{v}_i\| + d_{2i}\|\hat{v}_{ei}\|) \\ \leq & -\sum_{i=1}^{n}\left(\frac{1}{2}\hat{\eta}_i^T C_i K_i \hat{\eta}_i + \frac{1}{2}\hat{\psi}_i^T C_i \hat{\psi}_i\right) - \frac{k_2}{2}\|\hat{v}_e\|^2 - \frac{k_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\|\beta_i-\beta_j\|^2 \\ & + c_{7M}\|\tilde{v}\|^2 + d_{1M}\sqrt{n}\|\tilde{v}\| + d_{2M}\sqrt{n}\|\hat{v}_e\|, \end{aligned} \tag{46}$$

where $c_{7i}=c_{5i}+c_{6i}+\sum_{j=1}^{n} n\bar{c}_{6j}$, $c_{7M}=\max\{c_{7i}\}$, $\hat{v}_e=\mathrm{col}(\hat{v}_{e1},\hat{v}_{e2},\ldots,\hat{v}_{en})$, $\tilde{v}=\mathrm{col}(\tilde{v}_1,\tilde{v}_2,\ldots,\tilde{v}_n)$, d_{1i} and d_{2i} are some positive constants, $d_{1M}=\max\{d_{1i}\}$, $d_{2M}=\max\{d_{2i}\}$, and the inequalities $\sum_{i=1}^{n}\|\tilde{v}_i\|\leq\sqrt{n}\|\tilde{v}\|$ and $\sum_{i=1}^{n}\|\hat{v}_{ei}\|\leq\sqrt{n}\|\hat{v}_e\|$ have been used.



(a) $\omega_{i,1}$ (b) $\omega_{i,2}$ (c) $\omega_{i,3}$

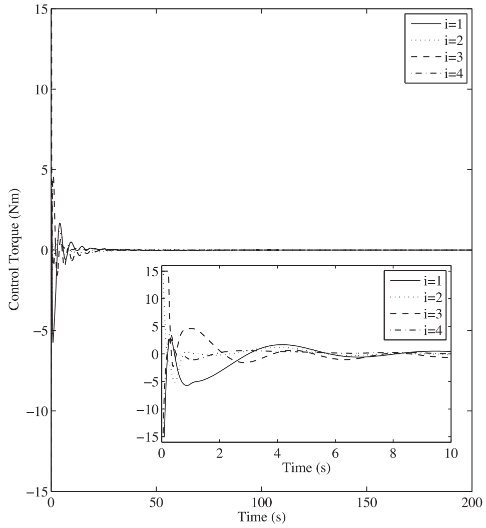
Fig. 2. Angular velocity (rad/s).

Using (24) and (46) and choosing $K > \max\{c_{7M}\theta^4, d_{1M}\sqrt{n}\theta^2\}$, the time derivative of V is obtained as

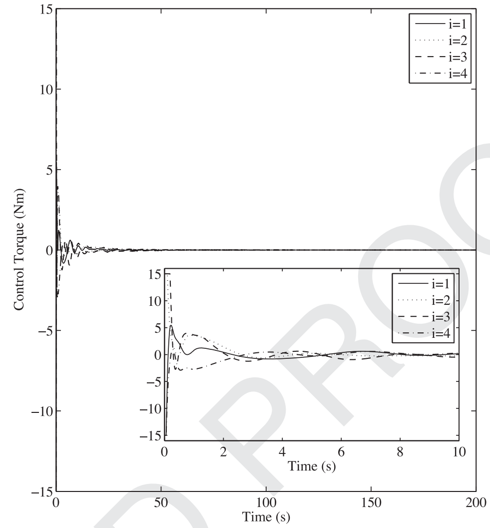
$$\dot{V} \leq -\sum_{i=1}^{n}\left(\frac{1}{2}\hat{\eta}_i^T C_i K_i \hat{\eta}_i + \frac{1}{2}\hat{\psi}_i^T C_i \hat{\psi}_i\right) - \|\hat{\nu}_e\|\left(\frac{k_2}{2}\|\hat{\nu}_e\| - d_{2M}\sqrt{n}\right) - \frac{k_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\|\beta_i - \beta_j\|^2 - \frac{K}{2}\|\varphi\|^2 - K\|\zeta\|((\bar{\theta} - 1)\|\zeta\| - 1), \tag{47}$$

where $\zeta = \mathrm{col}(\zeta_1, \zeta_2, \dots, \zeta_n)$ and $\varphi = \mathrm{col}(\varphi_1, \varphi_2, \dots, \varphi_n)$. Let $\bar{\theta} > 1$, and we obtain that $\dot{V} < 0$ if $\|\hat{\nu}_e\| > 2d_{2M}\sqrt{n}/k_2$ and $\|\zeta\| > 1/(\bar{\theta} - 1)$. Thus, we can conclude all signals in the closed-loop system are bounded, and $\lim_{t\to\infty} V(t)$ is also bounded. From Theorem 1 and Remark 1, it follows that $\tilde{\nu}_i(t) (i = 1, 2, \cdots, n)$ converges to zero exponentially as $t \to \infty$. Let V_{3i} be

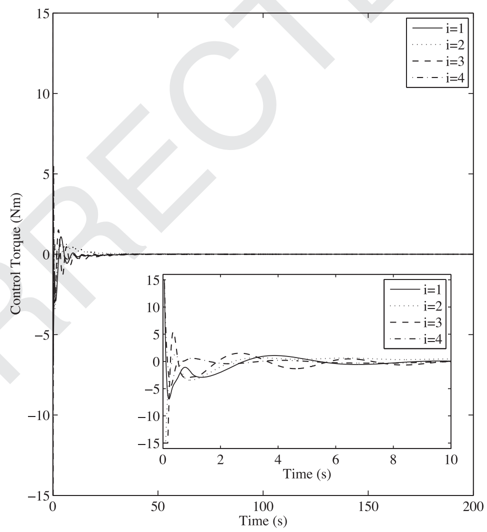
$$V_{3i} = \frac{1}{2}\hat{\nu}_{ei}^T \hat{\nu}_{ei} \tag{48}$$



(a) $\tau_{i,1}$



(b) $\tau_{i,2}$



(c) $\tau_{i,3}$

Fig. 3. Control torque.

and it has the derivative

$$\dot{V}_{3i} \leq -\frac{k_2}{2}\|\hat{v}_{ei}\|^2 + c_{6i}\|\tilde{v}_i\|^2 + \sum_{j=1}^{n}\bar{c}_{6j}\|\tilde{v}_j\|^2 \leq -k_2 V_{3i} + m_{1i}e^{-m_{2i}t}, \tag{49}$$

where m_{1i} and m_{2i} are some positive constants, and the inequality (45) has been used. Let $\hat{V}_{3i}(0) = V_{3i}(0)$ and

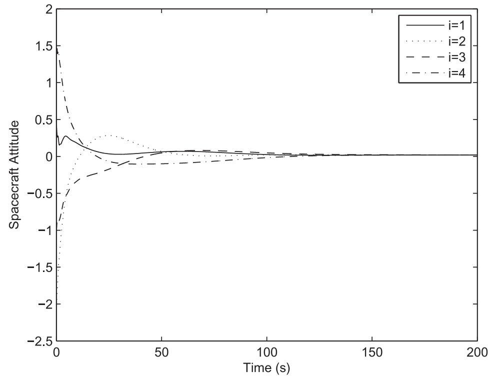
$$\dot{\hat{V}}_{3i} = -k_2\hat{V}_{3i} + m_{1i}e^{-m_{2i}t}. \tag{50}$$

The analytical solution of the preceding differential equation is given by

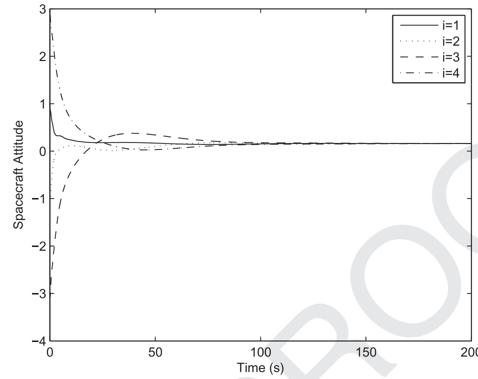
$$\hat{V}_{3i}(t) = m_{3i}e^{-k_2 t} + m_{4i}e^{-m_{2i}t}, \tag{51}$$

where $m_{3i} = \hat{V}_{3i}(0) + m_{1i}/(m_{2i} - k_2)$, and $m_{4i} = -m_{1i}/(m_{2i} - k_2)$. By the comparison lemma [11], it follows that $V_{3i}(t) \leq \hat{V}_{3i}(t) = m_{3i}e^{-k_2 t} + m_{4i}e^{-m_{2i}t}$. Therefore, $\hat{v}_{ei}(t)$ exponentially converges to zero as $t \to \infty$.

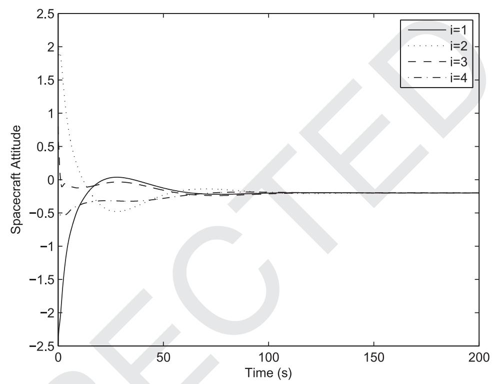
Please cite this article as: A.-M. Zou et al., Distributed attitude synchronization control for a group of flexible spacecraft using only attitude measurements, Information Sciences (2016), http://dx.doi.org/10.1016/j.ins.2016.01.048



(a) $q_{i,1}$



(b) $q_{i,2}$



(c) $q_{i,3}$

Fig. 4. Spacecraft attitude.

Since $\tilde{v}_i(t)$ and $\hat{v}_{ei}(t)$ exponentially converge to zero as $t \to \infty$, and all signals in the closed-loop system are bounded, the time derivative of V_1 in (38) becomes

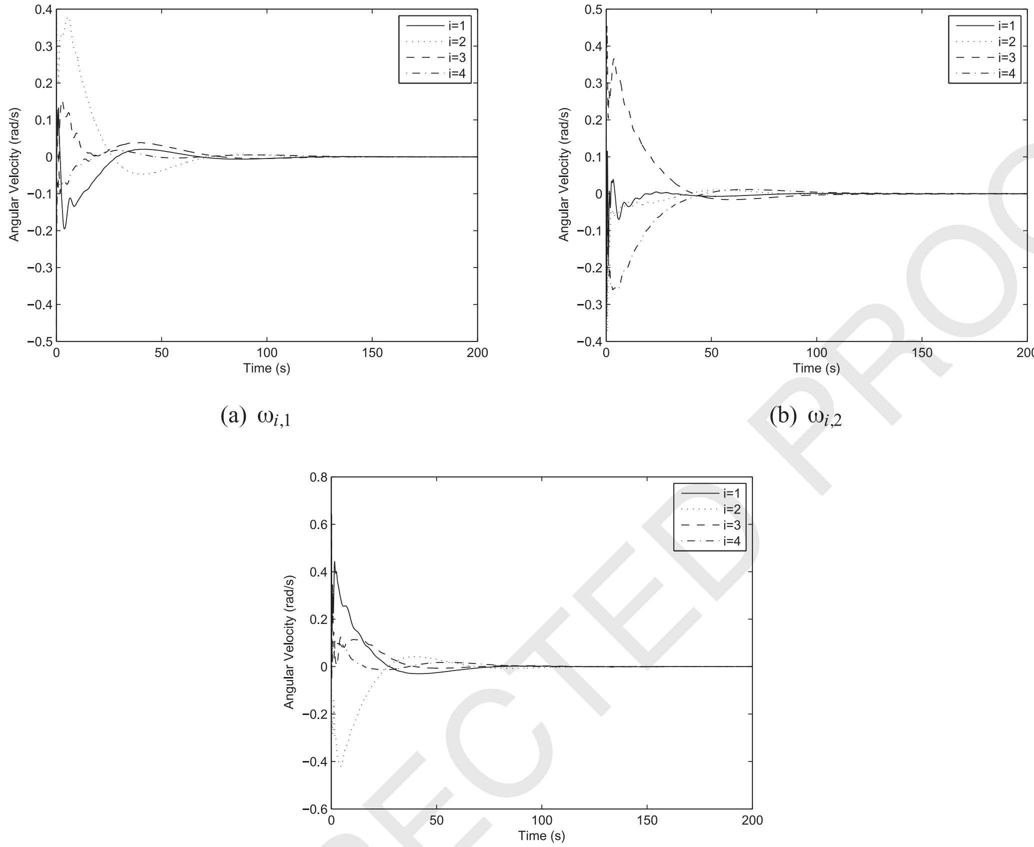
$$\begin{aligned} \dot{V}_1 &\leq -\sum_{i=1}^{n}\left(\frac{1}{2}\hat{\eta}_i^T C_i K_i \hat{\eta}_i + \frac{1}{2}\hat{\psi}_i^T C_i \hat{\psi}_i\right) - \frac{k_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\|\beta_i - \beta_j\|^2 + \sum_{i=1}^{n}(\beta_i^T \hat{v}_{ei} + \beta_i^T \tilde{v}_i + c_{5i}\|\tilde{v}_i\|^2) \\ &\leq -\sum_{i=1}^{n}\left(\frac{1}{2}\hat{\eta}_i^T C_i K_i \hat{\eta}_i + \frac{1}{2}\hat{\psi}_i^T C_i \hat{\psi}_i\right) - \frac{k_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\|\beta_i - \beta_j\|^2 + m_{5i}e^{-m_{6i}t}, \end{aligned} \tag{52}$$

where m_{5i} and m_{6i} are some positive constants. Integration of (52) from $t = 0$ to ∞ results in

$$\begin{aligned} &\int_{t=0}^{\infty}\sum_{i=1}^{n}\frac{1}{2}\left(\hat{\eta}_i^T C_i K_i \hat{\eta}_i + \hat{\psi}_i^T C_i \hat{\psi}_i\right)dt + \frac{k_1}{2}\int_{t=0}^{\infty}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\|\beta_i - \beta_j\|^2 dt \\ &\quad \leq V_1(0) - V_1(\infty) + \frac{m_{5i}}{m_{6i}} < \infty \end{aligned} \tag{53}$$

which indicates that $\hat{\eta}_i \in L_2$, $\hat{\psi}_i \in L_2$, and for any $a_{ij} > 0$, $\beta_i - \beta_j \in L_2$. Since all signals in the closed-loop system are bounded, we have $\hat{\eta}_i \in L_\infty$, $\hat{\psi}_i \in L_\infty$, $\dot{\hat{\eta}}_i \in L_\infty$, $\dot{\hat{\psi}}_i \in L_\infty$, and for any $a_{ij} > 0$, $\beta_i - \beta_j \in L_\infty$ and $\dot{\beta}_i - \dot{\beta}_j \in L_\infty$. Using Barbalat's lemma [16], we obtain that $\lim_{t\to\infty}\hat{\eta}_i(t) = 0$, $\lim_{t\to\infty}\hat{\psi}_i(t) = 0$, and $\lim_{t\to\infty}(\beta_i - \beta_j)(t) = 0$ for any $a_{ij} > 0$. Since the graph G is connected, there exists a path between any two distinct spacecraft. Thus, $\beta_i - \beta_j \to 0$ as $t \to \infty$, for all $i \neq j, i, j \in [1, 2, \ldots, n]$. By the definition of β_i, it follows that $q_i - q_j \to 0$ as $t \to \infty$, for all $i \neq j, i, j \in [1, 2, \ldots, n]$. This means that the attitude synchronization is achieved asymptotically. □

Remark 3. The velocity-free attitude coordination control for a group of rigid spacecraft has been studied in [1,12,15,27]. It is worthy of mentioning that the control schemes presented in these works cannot be applied for a group of flexible



(a) $\omega_{i,1}$ (b) $\omega_{i,2}$

(c) $\omega_{i,3}$

Fig. 5. Angular velocity (rad/s).

spacecraft because of the effect of the flexible dynamics. Hence, it is highly desirable to design a new control law for attitude coordination control of a group of flexible spacecraft.

Remark 4. In [7], a quaternion-based distributed attitude coordination control law was proposed for a group of flexible spacecraft with the requirement of angular velocity measurements. However, it is only shown that the vector part of the quaternion of each spacecraft in the group can achieve synchronization, and it is not clear whether the true attitude synchronization can be achieved.

4. Simulation results

In this section, numerical simulations of the nonlinear equations of motion (2) and (7) together with the nonlinear observer (9) and (10) are presented to verify the effectiveness of the proposed controller (40). In the simulation, we consider a scenario where there are four flexible spacecraft. The model parameters are chosen as the same as those in [7]. The weights of the undirected matrix are $a_{12} = a_{21} = a_{23} = a_{32} = a_{34} = a_{43} = a_{41} = a_{14} = 0.4$, and $a_{ij} = 0$ for other elements. The limit on the control torque is 15 Nm. The controller and observer parameters are set as $k_1 = 0.5, k_2 = 15, \theta = 5, \theta_1 = 2$ and $\theta_2 = 5$. The initial conditions are $q_1(0) = [0.5, 1, -2.5]^T, q_2(0) = [-2, -1.2, 2]^T$, $q_3(0) = [-1, -3, 0.8]^T, q_4(0) = [1.5, 3, -0.5]^T, \omega_1(0) = [0.4, -0.1, 0.5]^T$, $\omega_2(0) = [0.2, -0.4, 0.1]^T, \omega_3(0) = [-0.5, 0.1, 0.6]^T$ and $\omega_4(0) = 0$. With the control law defined by (40), the attitudes and angular velocities of all four spacecraft are shown in Figs. 1 and 2, respectively. It is observed that the attitudes of all spacecraft can reach the same attitude simultaneously. The bounded control torque is depicted in Fig. 3.

Next, we examine the effect of parameter uncertainties on the performance of the controller. For this case, we assume the inertia matrix $J_i = J_{0i} + \Delta J_i$, where J_{0i} is the nominal value of the inertia matrix and $\Delta J_i = 0.1J_{0i}$ is the parameter perturbation of the inertia matrix. The nominal value of the inertia matrix is chosen as the same as the inertia matrix in [7]. The responses of spacecraft attitude and angular velocity are shown in Figs. 4 and 5. It is found that the attitude synchronization can be still achieved even in the presence of parameter uncertainties.

Please cite this article as: A.-M. Zou et al., Distributed attitude synchronization control for a group of flexible spacecraft using only attitude measurements, Information Sciences (2016), http://dx.doi.org/10.1016/j.ins.2016.01.048

5. Conclusions

Based on a nonlinear observer, and the backstepping method, a distributed attitude synchronization control law was proposed for a group of flexible spacecraft without requiring the measurements of both the spacecraft angular velocities and modal variables. The performance of the proposed controller was evaluated through numerical simulations of the governing nonlinear system equations of motion. It was shown that the proposed controller can guarantee the attitudes of all spacecraft asymptotically converging to the same attitude. It is important to point out that by removing the requirement of both modal variables and spacecraft angular velocity measurements in the proposed control scheme, the cost related to the onboard sensors can be reduced. In the present paper, we assume that the communication graph among spacecraft is fixed and undirected. But the case where the communication graph is time varying may occur in practical applications. In future work, it will be interesting to investigate the problem of attitude coordination control for a group of flexible spacecraft under time-varying and directed communication graph.

References

[1] A. Abdessameud, A. Tayebi, Attitude synchronization of a group of spacecraft without velocity measurements,, IEEE Trans. Autom. Control 54 (11) (2009) 2642–2648.
[2] H. Cai, J. Huang, The leading-following attitude control of multiple rigid spacecraft systems,, Automatica 50 (4) (2014) 1109–1115.
[3] F. Celani, A luenberger-style observer for robot manipulators with position measurements,, in: Proceedings of the 14th Mediterranean Conference on Control and Automation, 2006, pp. 1–6.
[4] I. Changa, S.Y. Park, K.H. Choi, Decentralized coordinated attitude control for satellite formation flying via the state-dependent Riccati equation technique,, Int. J. Non-Linear Mech. 44 (8) (2009) 891–904.
[5] S.J. Chung, U. Ahsun, J.J.E. Slotine, Application of synchronization to formation flying spacecraft: Lagrangian approach,, J. Guid. Control Dyn. 32 (2) (2009) 512–526.
[6] S.D. Gennaro, Output stabilization of flexible spacecraft with active vibration suppression,, IEEE Trans. Aerosp. Electron. Syst. 39 (3) (2003) 747–759.
[7] H. Du, S. Li, Attitude synchronization control for a group of flexible spacecraft,, Automatica 50 (2) (2014) 646–651.
[8] H. Du, S. Li, C. Qian, Finite-time attitude tracking control of spacecraft with application to attitude synchronization,, IEEE Trans. Autom. Control 56 (11) (2011) 2711–2717.
[9] E. Jin, X. Jiang, Z. Sun, Robust decentralized attitude coordination control of spacecraft formation,, Syst. Control Lett. 57 (7) (2008) 567–577.
[10] I. Kanellakopoulos, P.V. Kokotovic, A.S. Morse, Systematic design of adaptive controllers for feedback linearizable systems,, IEEE Trans. Autom. Control 36 (11) (1991) 1241–1253.
[11] H.K. Khalil, Nonlinear Systems, third, Upper Saddle River, NJ: Prentice Hall, 2002.
[12] J. Lawton, R.W. Beard, Synchronized multiple spacecraft rotations,, Automatica 38 (8) (2002) 1359–1364.
[13] Z. Meng, W. Ren, Z. You, Distributed finite-time attitude containment control for multiple rigid bodies,, Automatica 46 (12) (2010) 2092–2099.
[14] W. Ren, Distributed attitude alignment in spacecraft formation flying,, Int. J. Adapt. Control Signal Process. 21 (2–3) (2007) 95–113.
[15] W. Ren, Distributed cooperative attitude synchronization and tracking for multiple rigid bodies,, IEEE Trans. Control Syst. Technol. 18 (2) (2010) 383–392.
[16] S. Sastry, M. Bodson, Adaptive Control: Stability, Convergence, and Robustness, Englewood Cliffs, NJ: Prentice-Hall, 1989.
[17] K. Shirai, Y. Amano, Application of an autonomous distributed system to the production process,, Int. J. Innov. Comput. Inf. Control 10 (4) (2014) 1247–1265.
[18] M.D. Shuster, A survey of attitude representations,, J. Astronaut. Sci. 41 (4) (1993) 439–517.
[19] X. Su, L. Wu, P. Shi, Sensor networks with random link failures: distributed filtering for TCS fuzzy systems,, IEEE Trans. Ind. Inf. 9 (3) (2013) 1739–1750.
[20] X. Su, L. Wu, P. Shi, Y.D. Song, A novel approach to output feedback control of fuzzy stochastic systems,, Automatica 50 (12) (2014) 3268–3275.
[21] M.C. VanDyke, C.D. Hall, Decentralized coordinated attitude control within a formation of spacecraft,, AIAA J. Guid. Control Dyn. 29 (5) (2006) 1101–1109.
[22] L. Wu, X. Yang, F. Li, Nonfragile output tracking control of hypersonic air-breathing vehicles with an LPV model,, IEEE/ASME Trans. Mechatron. 18 (4) (2013) 1280–1288.
[23] J. Zhou, Q. Hu, M.I. Friswell, Decentralized finite time attitude synchronization control of satellite formation flying,, AIAA J. Guid. Control. Dyn. 36 (1) (2013) 185–195.
Q5 [24] N. Zhou, Y. Xia, M. Wang, M. Fu, Finite-time attitudecontrol of multiple rigid spacecraft using terminal sliding mode,, Int. J. Robust Nonlinear Control (2014), doi:10.1002/rnc.3182.
[25] A.M. Zou, K.D. Kumar, Distributed attitude coordination control for spacecraft formation flying,, IEEE Trans. Aerosp. Electron. Syst. 48 (2) (2012a) 1329–1346.
[26] A.M. Zou, K.D. Kumar, Neural network-based distributed attitude coordination control for spacecraft formation flying with input saturation,, IEEE Transactions on Neural Networks and Learning Systems 23 (7) (2012b) 1155–1162.
[27] A.M. Zou, K.D. Kumar, Z.G. Hou, Attitude coordination control for a group of spacecraft without velocity measurements,, IEEE Transactions on Control Systems Technology 20 (5) (2012) 1160–1174.

Please cite this article as: A.-M. Zou et al., Distributed attitude synchronization control for a group of flexible spacecraft using only attitude measurements, Information Sciences (2016), http://dx.doi.org/10.1016/j.ins.2016.01.048