Distributed finite-time velocity-free attitude coordination control for spacecraft formations

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Abstract

In this paper, the finite-time velocity-free attitude coordination control for spacecraft formation flying under an undirected communication graph is addressed. A finite-time observer is introduced to obtain an accurate estimation of unmeasurable angular velocity and a decentralized finite-time observer is employed to estimate the angular acceleration of the virtual leader. With the application of the finite-time observer, the decentralized finite-time observer, and the homogeneous method, a continuous distributed finite-time attitude coordination control law is designed for a group of spacecraft without requiring angular velocity measurements. A rigorous proof shows that semi-global finite-time stability of the overall closed-loop system can be achieved and the proposed velocity-free control law guarantees a group of spacecraft to simultaneously track a common time-varying reference attitude in finite time even when the reference attitude is available only to a subset of the group members. The performance of the control scheme derived here is illustrated through numerical simulations.

Key words: Attitude coordination control; spacecraft formation flying; finite-time control; velocity-free; homogeneous method.

1 Introduction

In recent years, attitude coordination control for spacecraft formation flying (SFF) has attracted significant attention. This is because SFF is an applicable technology for many space missions such as Earth monitoring, geodesy, deep space imaging and exploration, and in-orbit servicing and maintenance of spacecraft.

A class of decentralized coordination tracking control laws was developed in VanDyke and Hall (2006). Ren (2007) proposed control laws for a team of spacecraft through local information exchange. With consideration of external disturbances and time delays, Jin, Jiang, and Sun (2008) presented a decentralized variable structure controller for attitude coordination control of multiple spacecraft. Using a state-dependent Riccati equation technique, Chang, Park, and Choi (2009) proposed a decentralized attitude coordination control algorithm for satellite formation flying. Chung, Ahsun, and Slotine (2009) employed a Lagrangian approach and nonlinear contraction analysis to study the problem of cooperative tracking control for SFF. Cai and Huang (2014) studied the leader-follower attitude consensus problem for a multiple rigid spacecraft system. In these works, the control laws require full state measurements. Based on a bi-directional ring topology, Lawton and Beard (2002) proposed a passivity based formation control law for multi-spacecraft attitude alignment. Later, Ren (2010) extended the work of Lawton and Beard (2002) to the case of a general undirected connected communication topology. In Lawton and Beard (2002) and Ren (2010), the case when the final angular velocity is zero is considered, and the extension of the obtained results to the attitude consensus tracking is not straightforward. Abdessameud and Tayebi (2009) proposed a velocity-free attitude tracking and synchronization control scheme for a group of spacecraft. However, the common time-varying reference attitude was assumed to be available to each spacecraft in the group, which implies that there exists a central station or a leader which can not only obtain the group reference but also communicate with each group member in the formation. The requirement of such a leader introduces an apparent limitation and the information relay will result in increased complexity especially when there are a large number of spacecraft. Therefore, in practical applications, it may be more realistic that a common time-varying reference attitude is available only to a subset of the group members. A velocity-free attitude coordination control scheme has been designed for such a group of spacecraft in (Zou,

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Kumar and Hou, 2012).

The aforementioned attitude coordination laws achieve asymptotic stability with infinite convergence time. In the SFF attitude coordinated control, the finite-time control implies faster formation rearrangement capability, which leads to an enhanced application efficiency of SFF. Finite-time attitude control for a single spacecraft has been studied in Jin and Sun (2008), Zhu, Xia, and Fu (2011), Zou et al. (2011), Du, Li, and Qian (2011), Du and Li (2012, 2013), Lu and Xia (2013) and Zou (2014). However, the extension of the finite-time attitude control algorithms from the single spacecraft case to the multiple spacecraft case is nontrivial especially for the case when there exists a dynamic (virtual) leader whose state is not accessible to all followers.

Several authors have investigated the finite-time attitude cooperative control problem, e.g., Meng, Ren, and You (2010), Du, Li, and Qian (2011), Zou and Kumar (2012), Zhou, Hu, and Friswell (2013), and Zhou et al. (2014). In Meng, Ren, and You (2010), based on a distributed sliding-mode estimator and a nonsingular sliding mode surface, a distributed finite-time control law was designed for a group of rigid bodies with a dynamic leader. However, the control law is discontinuous, and the discontinuity of the control input may cause chattering behavior and excite unmodeled high-frequency system dynamics. Du, Li, and Qian (2011) proposed a distributed finite-time attitude control scheme for SFF under a communication graph which has a hierarchical structure. However, the control law is not applicable to finite-time attitude coordination control for SFF under an undirected communication graph. In Zhou, Hu, and Friswell (2013), a quaternion-based finite-time attitude coordination control law was proposed for satellite formation flying. However, it is only shown that the vector part of the quaternion of each member in the group can track the desired trajectory, and it is not clear whether the attitude synchronization and tracking can be achieved in finite time. Based on the adaptive sliding mode control technique, decentralized finite-time attitude control laws were proposed for multiple rigid spacecraft in Zhou et al. (2014). However, each spacecraft in the formation has its own reference trajectory, and the control laws are not extendable to the case when there is a common time-varying reference attitude which is available to only a subset of the group members. Furthermore, the aforementioned cooperative finite-time attitude control laws rely on the availability of angular velocity measurements. However, in practical applications, due to either cost limitations or implementation considerations, angular velocity measurements may not be available. Therefore, it is highly desirable to design a velocity-free distributed attitude coordination control law that can provide finite-time control for SFF.

In this paper, we study finite-time velocity-free attitude coordination control for SFF under an undirected communication graph. We consider the case when the common time-varying reference attitude is available only to a subset of the team members. The attitude of each spacecraft in the formation is represented by modified Rodrigues parameters (MRPs). Using the finite-time observer introduced in Zou (2014), a decentralized finite-time observer and the homogeneous method, we propose a distributed semi-global velocity-free finite-time attitude coordination control law for SFF. In this paper, the term "semi-global stability" refers to the attitude system using MRPs-based representation. The proposed control law is useful and valuable during formation acquisition and deployment phases.

2 Background and Preliminaries

2.1 Notation, Definitions and Lemmas

The notation $\|\cdot\|$ refers to the Euclidean norm of a vector or the induced norm of a matrix. I_n represents the $n \times n$ identity matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of a matrix, respectively. The Kronecker product is denoted by $\otimes$. Given a vector $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ and $\alpha \in R$, define $x^\alpha = [x_1^\alpha, x_2^\alpha, \cdots, x_n^\alpha]^T$, $\text{sig}^\alpha(x) = [\text{sgn}(x_1)|x_1|^\alpha, \text{sgn}(x_2)|x_2|^\alpha, \cdots, \text{sgn}(x_n)|x_n|^\alpha]^T$, and $\text{diag}(|x|^\alpha) = \text{diag}(|x_1|^\alpha, |x_2|^\alpha, \cdots, |x_n|^\alpha)$, where $\text{sgn}(\cdot)$ denotes the signum function defined by $\text{sgn}(y) = 1$ if $y \geq 0$ and $\text{sgn}(y) = -1$ if $y < 0$, $\forall y \in R$. For any $\lambda > 0$ and any set of real parameters $r_i > 0 (i = 1, 2, \cdots, n)$, a dilation operator $\delta_\lambda^r : R^n \longmapsto R^n$ is defined by $\delta_\lambda^r(x_1, x_2, \cdots, x_n) = (\lambda^{r_1} x_1, \lambda^{r_2} x_2, \cdots, \lambda^{r_n} x_n)$, where $r = [r_1, r_2, \cdots, r_n]^T$.

Definition 1 (Nakamura, Yamashita, & Nishitani, 2004). A continuous function $f : R^n \longmapsto R$ is homogeneous of degree k with respect to the dilation δ_λ^r if $\forall \lambda > 0, f(\delta_\lambda^r(x)) = \lambda^k f(x)$, where $k > -\min\{r_i\}, i = 1, 2, \cdots, n$. A differential system $\dot{x} = f(x)$ (or a vector field f), with continuous $f : R^n \longmapsto R^n$, is homogeneous of degree k with respect to the dilation δ_λ^r if $\forall \lambda > 0, f_i(\delta_\lambda^r(x)) = \lambda^{k+r_i} f_i(x), i = 1, 2, \cdots, n$.

Definition 2 (Hong, Wang, & Cheng, 2006). Consider the following system:

$$\dot{x} = f(x,t), f(0,t) = 0, x \in U \subset R^n \tag{1}$$

where $f : U \times R^+ \to R^n$ is continuous on an open neighborhood U of the origin $x = 0$. The zero solution of (1) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U_0 \subseteq U$ of the origin. The "finite-time convergence" means: If, for any initial condition $x(t_0) = x_0 \in U_0$ at any given initial time t_0, there is a setting time $T > 0$, such that every solution $x(t; t_0, x_0)$ of system (1) is defined with $x(t; t_0, x_0) \in U_0 \backslash \{0\}$ for $t \in [t_0, T)$, $\lim_{t \to T} x(t; t_0, x_0) =$

0, and $x(t;t_0,x_0)=0, \quad \forall t>T$. When $U=U_0=R^n$, the zero solution is said to be globally finite-time stable.

Lemma 1 (Hong, Wang, & Cheng, 2006). Suppose that there is a Lyapunov function $V(x,t)$ defined on $U_1\times R^+$, where $U_1\subseteq U\in R^n$ is a neighborhood of the origin, and

$$\dot V(x,t)\le -lV^a(x,t), \forall x\in U_1\setminus\{0\} \tag{2}$$

where $l>0$ and $0<a<1$. Then, the origin of system (1) is locally finite-time stable. The settling time satisfies $T\le \frac{V^{1-a}(x(t_0),t_0)}{l(1-a)}$ for a given initial condition $x(t_0)\in U_1$.

Corollary 1. Suppose that there is a Lyapunov function $V(x,t)$ defined on $U_1\times R^+$, where $U_1\subseteq U\in R^n$ is a neighborhood of the origin, and

$$\dot V(x,t)\le -lV^a(x,t)+kV(x,t), \forall x\in U_1\setminus\{0\} \tag{3}$$

where $l,k>0$ and $0<a<1$. Then, for a given initial condition $x(t_0)$ at any initial time t_0, the origin of system (1) is locally finite-time stable if $x(t_0)\in\{U_1\cap U_2\}$, where $U_2=\{x|V^{1-a}(x,t)<l/k\}$ is a neighborhood of the origin and satisfies that $U_2\subseteq U_1$ or $U_1\subseteq U_2$. The settling time satisfies $T\le\frac{V^{1-a}(x(t_0),t_0)}{(l-kV(x(t_0),t_0))(1-a)}$ for a given initial condition $x(t_0)\in\{U_1\cap U_2\}$.

Proof. Note that if $x\in\{U_1\cap U_2\}$, then we have

$$\begin{aligned}\dot V(x,t)&\le -lV^a(x,t)+kV(x,t)\\ &=-(l-kV^{1-a}(x,t))V^a(x,t)\le 0\end{aligned} \tag{4}$$

which implies that $V(x,t)\le V(x(t_0),t_0)$ for any initial condition $x(t_0)\in\{U_1\cap U_2\}$. Thus, (4) becomes

$$\dot V(x,t)\le -(l-kV^{1-a}(x(t_0),t_0))V^a(x,t). \tag{5}$$

The conclusion follows from Lemma 1. □

Lemma 2 (Qian & Lin, 2001). For any $x\in R, y\in R, c>0, d>0$, and $\gamma>0$, $|x|^c|y|^d\le c\gamma|x|^{c+d}/(c+d)+d|y|^{c+d}/[\gamma^{c/d}(c+d)]$.

Lemma 3 (Hardy, Littlewood, & Polya, 1952). For any $x_i\in R, i=1,2,\cdots,n$, and a real number $\nu\in(0,1]$, $(\sum_{i=1}^n|x_i|)^\nu\le\sum_{i=1}^n|x_i|^\nu\le n^{1-\nu}\left(\sum_{i=1}^n|x_i|\right)^\nu$.

Lemma 4. For any $x\in R, y\ne 0\in R, c>0, d>0$, and $c-d>0$, the following inequality holds:

$$\frac{|x|^c}{|y|^d}\ge\frac{c}{c-d}|x|^{c-d}-\frac{d}{c-d}|y|^{c-d}. \tag{6}$$

Proof. By Lemma 2, we have $|x|^{c-d}|y|^d\le\frac{c-d}{c}|x|^c+\frac{d}{c}|y|^c$, which implies that $c|x|^{c-d}\le(c-d)\frac{|x|^c}{|y|^d}+d|y|^{c-d}$, which rearranges to give (6). □

Lemma 5. For any $x_i\in R, i=1,2,\cdots,n$, and a real number $p>1$, $\sum_{i=1}^n|x_i|^p\le(\sum_{i=1}^n|x_i|)^p\le n^{p-1}\sum_{i=1}^n|x_i|^p$.

Proof. We consider two cases, i.e., $\sum_{i=1}^n|x_i|=0$ and $\sum_{i=1}^n|x_i|\ne0$. It is clear that the lemma holds for $\sum_{i=1}^n|x_i|=0$. For $\sum_{i=1}^n|x_i|\ne0$, we first show $\sum_{i=1}^n|x_i|^p\le(\sum_{i=1}^n|x_i|)^p$. By noticing that $p>1$, we obtain

$$\sum_{i=1}^n\left(\frac{|x_i|}{\sum_{j=1}^n|x_j|}\right)^p\le\sum_{i=1}^n\frac{|x_i|}{\sum_{j=1}^n|x_j|}=1. \tag{7}$$

Multiplying both sides of (7) by $\left(\sum_{j=1}^n|x_j|\right)^p$, we can obtain that $\sum_{i=1}^n|x_i|^p\le(\sum_{i=1}^n|x_i|)^p$. By Lemma 4,

$$n^{p-1}\left(\frac{|x_i|}{\sum_{j=1}^n|x_j|}\right)^p\ge\frac{p|x_i|}{\sum_{j=1}^n|x_j|}-(p-1)\frac{1}{n} \tag{8}$$

leading to

$$\begin{aligned}n^{p-1}\sum_{i=1}^n\left(\frac{|x_i|}{\sum_{j=1}^n|x_j|}\right)^p&\ge p\frac{\sum_{i=1}^n|x_i|}{\sum_{j=1}^n|x_j|}-(p-1)\\ &=1\end{aligned} \tag{9}$$

Multiplying both sides of (9) by $\left(\sum_{j=1}^n|x_j|\right)^p$, we can obtain that $(\sum_{i=1}^n|x_i|)^p\le n^{p-1}\sum_{i=1}^n|x_i|^p$. This completes the proof of Lemma 5. □

2.2 Problem Formulation

Consider a group of n spacecraft in which the ith ($i=1,2,\cdots,n$) spacecraft is governed by (Hughes, 1986)

$$\dot q_i=T_i(q_i)\omega_i,\ J_i\dot\omega_i=-\omega_i^\times J_i\omega_i+\tau_i \tag{10}$$

where $J_i\in R^{3\times3}$ is the inertia matrix, $\tau_i\in R^3$ is the torque control, $\omega_i\in R^3$ is the angular velocity of the ith spacecraft in a body-fixed frame, $q_i(t)\in R^3$ represents the MRPs (Shuster, 1993) describing the spacecraft attitude with respect to an inertial frame, defined by $q_i(t)=\varrho_i(t)\tan\left(\frac{\phi_i(t)}{4}\right)$, $\phi_i\in[0,2\pi)$rad with ϱ_i and ϕ_i denoting the Euler eigenaxis and eigenangle, respectively. $a^\times\in R^{3\times3}$ denotes a 3×3 skew-symmetric matrix such that $a\times b=a^\times b$ for any $a,b\in R^3$. The matrix $T_i(q_i) \in R^{3\times 3}$ is given by (Shuster, 1993)

$$T_i(q_i) = \frac{1}{2}\left[\frac{1-q_i^T q_i}{2} I_3 + q_i^\times + q_i q_i^T\right]. \tag{11}$$

Equation (10) can be transformed as

$$\dot{q}_i = v_i, \quad \dot{v}_i = f_i(q_i, v_i) + g_i(q_i)\tau_i \tag{12}$$

where $v_i = \dot{q}_i$, $f(q_i, v_i) = -T_i \dot{P}_i \dot{q}_i - T_i J_i^{-1}(P_i \dot{q}_i)^\times J_i P_i \dot{q}_i$, $P_i = T_i^{-1}(q_i)$, and $g_i = T_i J_i^{-1}$.

The main objective of the present work is to design a distributed control law for $\tau_i (i = 1, 2, \cdots, n)$ such that the attitude state of all spacecraft in the formation can track a time-varying reference trajectory in finite time even in the absence of angular velocity measurement. Furthermore, the control law developed in this paper is directly applicable to any second-order multi-agent system in the form of (12).

We model the topology of the information flow among spacecraft by a weighted undirected connected graph $G = (\Upsilon, E, A)$, where $\Upsilon = \{\varsigma_1, \varsigma_2, \cdots, \varsigma_n\}$ is the set of nodes, $E \subseteq \Upsilon \times \Upsilon$ is the set of edges, and $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of graph G with nonnegative elements. Node $\varsigma_i (i = 1, 2, \cdots, n)$ represents the ith spacecraft, and an edge in G is denoted by an unordered pair $(\varsigma_i, \varsigma_j)$. $(\varsigma_i, \varsigma_j) \in E$ if and only if there is an information exchange between the ith spacecraft and the jth spacecraft. Since the graph is undirected, $(\varsigma_i, \varsigma_j) \in E \Leftrightarrow (\varsigma_j, \varsigma_i) \in E$. The adjacency element a_{ij} denotes the communication quality between the ith spacecraft and the jth spacecraft, i.e., $(\varsigma_i, \varsigma_j) \in E \Leftrightarrow a_{ij} > 0$. It is assumed that $a_{ij} = a_{ji}$ and $a_{ii} = 0$; that is, the weighted adjacency matrix A is a symmetric matrix.

Let $D = \text{diag}\{d_1, d_2, \cdots, d_n\}$ denote the degree matrix of the weighted graph G, whose diagonal element is given by $d_i = \sum_{j=1}^n a_{ij} (i = 1, 2, \cdots, n)$. The Laplacian matrix L of the weighted graph G is defined as $L = D - A$, which is a symmetric matrix. For any two nodes ς_i and ς_j, if there exists a path between them, then G is called a connected graph.

For the attitude consensus tracking control, we assume that there exists a virtual leader, labeled as spacecraft 0 and its state is given by $q_0 \in R^3$, a time-varying reference trajectory for the spacecraft formation. We use the graph $\bar{G}$ to model the network topology associated with the system consisting of n spacecraft and one virtual leader. Let $B = \text{diag}\{a_{10}, a_{20}, \cdots, a_{n0}\}$ be the adjacency matrix associated with $\bar{G}$, where $a_{i0} > 0 (i = 1, 2, \cdots, n)$ is a constant if the ith spacecraft has access to the leader, otherwise $a_{i0} = 0$. For $\bar{G}$, if there is a path in $\bar{G}$ from the node ς_0 (leader) to every node $\varsigma_i (i = 1, 2, \cdots, n)$, then $\bar{G}$ is called a connected graph.

Theorem 1 (Hong, Hu, & Gao, 2006). If $\bar{G}$ is connected, then the matrix $L + B$ associated with $\bar{G}$ is symmetric and positive definite.

Assumption 1. The reference attitude q_0 and its first three derivatives satisfy $\|q_0\| \le B_1$, $\|\dot{q}_0\| \le B_2$, $\|\ddot{q}_0\| \le B_3$ and $\|\dddot{q}_0\| \le B_4$, where $B_i > 0 (i = 1, 2, 3, 4)$ are known constants.

Remark 1. The attitude consensus tracking problem has also been investigated in Abdessameud and Tayebi (2009), Cai and Huang (2014), Chung, Ahsun, and Slotine (2009), Ren (2010), and VanDyke and Hall (2006). In Abdessameud and Tayebi (2009), Chung, Ahsun, and Slotine (2009), and VanDyke and Hall (2006), the authors assume that the common reference trajectory is accessible to all group members. In Ren (2010), angular accelerations are assumed to be available for exchange among team members. In Cai and Huang (2014), the desired angular velocity is assumed to be generated by a linear autonomous system known to all spacecraft in the formation. In the present investigation these assumptions are not necessary.

3 Main Results

In this section, we focus on designing a distributed finite-time attitude coordination control law for SFF. We assume that only measurement of spacecraft attitude is available for use in designing the control law and that only a subset of the group members has access to the virtual leader. To solve this problem, we use a finite-time observer, a decentralized finite-time observer and the homogenous method to design a distributed control law such that all spacecraft in the group can simultaneously track a time-varying reference attitude in finite time even in the absence of angular velocity measurement and when the reference attitude is available to only a subset of the team members.

3.1 Finite-Time Observers

Since measurements of angular velocity for each spacecraft in the group are unavailable, the finite-time observer developed in Zou (2014) is introduced to obtain an accurate estimation of the angular velocity.

Lemma 6 (Zou, 2014). Consider the following observer for system (12):

$$\begin{cases} \dot{\hat{q}}_i = \hat{v}_i + \theta\beta_1 \text{sig}^\alpha(\tilde{q}_i) \\ \dot{\hat{v}}_i = f_i(q_i, \hat{v}_i) + g_i(q_i)\tau_i + \theta^2\beta_2 \text{sig}^{\alpha_1}(\tilde{q}_i) \end{cases} \tag{13}$$

where $i = 1, 2, \cdots, n$, $\hat{q}_i$ and $\hat{v}_i$ are estimates of q_i and v_i, respectively, $\hat{q}_i(0) = q_i(0)$, $\hat{v}_i(0) = 0$, $\tilde{q}_i = q_i - \hat{q}_i$, $\alpha \in (1/2, 1)$, $\alpha_1 = 2\alpha - 1$, θ, β_1, and β_2 are positive constants, respectively. For any given constant $\Delta > 0$, if $q_i, v_i, \hat{q}_i$ and $\hat{v}_i$ lie within the compact set $\Omega_1 = \{(q_i, v_i, \hat{q}_i, \hat{v}_i) | \|q_i\| \le \Delta, \|\hat{q}_i\| \le \Delta, \|v_i\| \le \Delta, \|\hat{v}_i\| \le \Delta\}$, then there exists a sufficiently large observer parameter θ such that system (13) admits semi-global finite-time observer.

To facilitate the stability analysis of the closed-loop system, a brief proof of this lemma is presented as follows.

Proof. Define $\epsilon_{1i} = \tilde{q}_i/\theta^{\theta_1}$, $\epsilon_{2i} = \tilde{v}_i/\theta^{1+\theta_1}$, $\tilde{\epsilon}_{1i} = \epsilon_{1i}$, $\tilde{\epsilon}_{2i} = \mathrm{sig}^{1/\alpha}(\epsilon_{2i})$, $\tilde{\epsilon}_i = [\tilde{\epsilon}_{1i}^T, \tilde{\epsilon}_{2i}^T]^T$, and the dilation $\delta_\lambda^r(\epsilon_{1i}, \epsilon_{2i}) = (\lambda\epsilon_{1i}^T, \lambda^\alpha\epsilon_{2i}^T)$, where $0 < \theta_1 < 1$, and $\tilde{v}_i = v_i - \hat{v}_i$. Let $M = \begin{bmatrix} -\beta_1 I_3 & I_3 \\ -\beta_2 I_3 & 0 \end{bmatrix}$. Since the matrix M is Hurwitz, there exists $N = N^T > 0$ such that $M^T N + NM = -I_6$. Consider the Lyapunov function:

$$V_{oi}(\epsilon_i) = \tilde{\epsilon}_i^T N \tilde{\epsilon}_i \tag{14}$$

where $\epsilon_i = [\epsilon_{1i}^T, \epsilon_{2i}^T]^T$. Following the similar analysis as that in Zou (2014), we can obtain that

$$\dot{V}_{oi} \le -c_{2i} V_{oi}^\beta + c_{3i} V_{oi} \tag{15}$$

where $c_{2i}(\alpha, \theta) = -\max_{\{z: V_\alpha(z)=1\}} L_{f_{\epsilon_i}} V_{oi}(z)$, c_{3i} is a positive constant, and $\beta = (1+\alpha)/2$, and we can conclude that system (13) admits semi-global finite-time observer, i.e., for any constant $\Delta > 0$, the observer errors $\tilde{q}_i$ and $\tilde{v}_i (i = 1, 2, \cdots, n)$ converges to zero in finite time. □

As the leader's information is only available to a subset of the group members, decentralized finite-time observers are designed for each spacecraft in the group to obtain an estimation of the acceleration $\dot{v}_0 = \ddot{q}_0$.

Lemma 7. Consider the following decentralized finite-time observer for the ith spacecraft:

$$\begin{aligned} \dot{p}_i = &-\beta_3 \mathrm{sig}^{\frac{2}{\alpha}-1}\left[\sum_{j=1}^n a_{ij}(p_i - p_j) + a_{i0}(p_i - \dot{v}_0)\right] \\ &- \beta_4 \mathrm{sgn}\left[\sum_{j=1}^n a_{ij}(p_i - p_j) + a_{i0}(p_i - \dot{v}_0)\right] \end{aligned} \tag{16}$$

where $i = 1, 2, \cdots, n$, p_i is an estimation of $\dot{v}_0$, $p_i(0) = 0$, $\beta_3 > 0$ and $\beta_4 > B_4 \ge \|\ddot{v}_0\|$ are positive constants, respectively. Then, the estimation error $\tilde{p}_i = p_i - \dot{v}_0$ converges to zero in finite time.

Proof. Noting that $\sum_{j=1}^n a_{ij}(p_i - p_j) + a_{i0}(p_i - \dot{v}_0) = \sum_{j=1}^n l_{ij}\tilde{p}_j + a_{i0}\tilde{p}_i$, where l_{ij} is the element of the graph Laplacian matrix, the dynamic equation for $\tilde{p}_i$ is $\dot{\tilde{p}}_i = -\beta_3 \mathrm{sig}^{\frac{2}{\alpha}-1}\left(\sum_{j=1}^n l_{ij}\tilde{p}_j + a_{i0}\tilde{p}_i\right) - \beta_4 \mathrm{sgn}\left(\sum_{j=1}^n l_{ij}\tilde{p}_j + a_{i0}\tilde{p}_i\right) - \ddot{v}_0$. Consider the Lyapunov function

$$V_1(\tilde{p}) = \frac{1}{2}\tilde{p}^T M_1 \tilde{p} \tag{17}$$

where $\tilde{p} = [\tilde{p}_1^T, \tilde{p}_2^T, \cdots, \tilde{p}_n^T]^T$, and $M_1 = (L + B) \otimes I_3$. The time derivative of $V_1(\tilde{p})$ is given by

$$\begin{aligned} \dot{V}_1 &\le -\beta_3 \tilde{p}^T M_1 \mathrm{sig}^{\frac{2}{\alpha}-1}(M_1\tilde{p}) - (\beta_4 - B_4)\|M_1\tilde{p}\| \\ &\le -(\beta_4 - B_4)\sqrt{\frac{2\lambda_{\min}(M_1^2)}{\lambda_{\max}(M_1)}} V_1^{1/2} = -c_4 V_1^{1/2} \end{aligned} \tag{18}$$

which implies that the estimation error $\tilde{p}_i = p_i - \dot{v}_0 (i = 1, 2, \cdots, n)$ converges to zero in finite time. □

Remark 2. Different decentralized finite-time observers have been proposed in Meng, Ren and You (2010) and Li, Du and Lin (2011). In these works, asymptotic control laws are used to ensure the boundedness of the agents' states before the convergence of the decentralized finite-time observer, while finite-time control laws are applied to guarantee the finite-time stability after the convergence of the decentralized finite-time observer. Differing with these works, an extra term $-\beta_3 \mathrm{sig}^{\frac{2}{\alpha}-1}\left[\sum_{j=1}^n a_{ij}(p_i - p_j) + a_{i0}(p_i - \dot{v}_0)\right]$ is adopted in (16) to guarantee the stability of the closed-loop system even before the estimation error $\tilde{p}_i = p_i - \dot{v}_0 (i = 1, 2, \cdots, n)$ converges to zero.

3.2 Design of Finite-Time Controller

For the attitude consensus tracking control, there are two attitude state error measures for a group of spacecraft; that is, the station-keeping and formation-keeping attitude state errors. The station-keeping error is the attitude state error of an individual spacecraft in the formation with respect to the reference attitude state of the spacecraft formation, and the formation-keeping error is the attitude state error of an individual spacecraft with respect to the other spacecraft in the formation. The lumped attitude state error including station-keeping and formation-keeping errors for the ith spacecraft in the formation are defined as $\chi_{1i} = \sum_{j=1}^n a_{ij}(q_i - q_j) + a_{i0}(q_i - q_0)$ and $\chi_{2i} = \sum_{j=1}^n a_{ij}(v_i - v_j) + a_{i0}(v_i - v_0)$, where $v_0 = \dot{q}_0$, and $i = 1, 2, \cdots, n$. The lumped attitude state errors χ_{1i} and χ_{2i} can be reexpressed in terms of the station-keeping attitude state errors $e_{1i} = q_i - q_0$ and $e_{2i} = v_i - v_0$ as $\chi_{1i} = \sum_{j=1}^n l_{ij}e_{1j} + a_{i0}e_{1i}$ and $\chi_{2i} = \sum_{j=1}^n l_{ij}e_{2j} + a_{i0}e_{2i}$. Note that the error signal $\chi_{2i}(i = 1, 2, \cdots, n)$ cannot be used to design the control law as it depends on the velocity information. Instead, for each follower spacecraft, we introduce a new error signal $\hat{\chi}_{2i}(i = 1, 2, \cdots, n)$ as $\hat{\chi}_{2i} = \sum_{j=1}^n a_{ij}(\hat{v}_i - \hat{v}_j) +$

$a_{i0}(\hat{v}_i - v_0) = \sum_{j=1}^{n} l_{ij}\hat{e}_{2j} + a_{i0}\hat{e}_{2i}$, where $\hat{e}_{2i} = \hat{v}_i - v_0$ which is governed by the following dynamic equation:

$$\dot{\hat{e}}_{2i} = f_i(q_i, \hat{v}_i) + g_i(q_i)\tau_i + \theta^2\beta_2\text{sig}^{\alpha_1}(\tilde{q}_i) - \dot{v}_0. \quad (19)$$

Now, the distributed velocity-free finite-time control law for the ith spacecraft is chosen as follows:

$$\tau_i = g_i^{-1}[-k_1^2 k_2 \text{sig}^{\alpha_1}(\chi_{1i}) - k_1 k_3 \text{sig}^{\alpha_2}(\hat{\chi}_{2i}) - f_i(q_i, \hat{v}_i) - \theta^2\beta_2\text{sig}^{\alpha_1}(\tilde{q}_i) + p_i], \quad i = 1, 2, \cdots, n \quad (20)$$

where $\alpha_2 = \alpha_1/\alpha$, k_1, k_2 and k_3 are positive constants.

Substituting the control law (20) into (19) yields

$$\dot{\hat{e}}_{2i} = -k_1^2 k_2 \text{sig}^{\alpha_1}(\chi_{1i}) - k_1 k_3 \text{sig}^{\alpha_2}(\hat{\chi}_{2i}) + \tilde{p}_i. \quad (21)$$

Define $e_1 = [e_{11}^T, e_{12}^T, \cdots, e_{1n}^T]^T$, $e_2 = [e_{21}^T, e_{22}^T, \cdots, e_{2n}^T]^T$, $\hat{e}_2 = [\hat{e}_{21}^T, \hat{e}_{22}^T, \cdots, \hat{e}_{2n}^T]^T$, $\tilde{e}_2 = [\tilde{e}_{21}^T, \tilde{e}_{22}^T, \cdots, \tilde{e}_{2n}^T]^T$, $\chi_1 = [\chi_{11}^T, \chi_{12}^T, \cdots, \chi_{1n}^T]^T$, and $\hat{\chi}_2 = [\hat{\chi}_{21}^T, \hat{\chi}_{22}^T, \cdots, \hat{\chi}_{2n}^T]^T$, where $\tilde{e}_{2i} = e_{2i} - \hat{e}_{2i} = \tilde{v}_i (i = 1, 2, \cdots, n)$. Then we have $\chi_1 = M_1 e_1$ and $\hat{\chi}_2 = M_1\hat{e}_2$ which are governed by

$$\dot{\chi}_1 = M_1 e_2 = \hat{\chi}_2 + M_1\tilde{e}_2 = \hat{\chi}_2 + M_1\tilde{v} \quad (22)$$

$$\dot{\hat{\chi}}_2 = -k_1^2 k_2 M_1 \text{sig}^{\alpha_1}(\chi_1) - k_1 k_3 M_1 \text{sig}^{\alpha_2}(\hat{\chi}_2) + M_1\tilde{p} \quad (23)$$

where $\tilde{v} = [\tilde{v}_1^T, \tilde{v}_2^T, \cdots, \tilde{v}_n^T]^T$. Define $\varphi_1 = \chi_1$, $\varphi_2 = \hat{\chi}_2/k_1$, and $\epsilon_2 = [\epsilon_{21}^T, \epsilon_{22}^T, \cdots, \epsilon_{2n}^T]^T$, then we obtain

$$\dot{\varphi}_1 = k_1\varphi_2 + M_1\tilde{v} = k_1\varphi_2 + \theta^{1+\theta_1}M_1\epsilon_2 \quad (24)$$

$$\dot{\varphi}_2 = -k_1 k_2 M_1 \text{sig}^{\alpha_1}(\varphi_1) - k_1^{\alpha_2} k_3 M_1 \text{sig}^{\alpha_2}(\varphi_2) + \frac{M_1\tilde{p}}{k_1}. \quad (25)$$

Let $\bar{M} = \begin{bmatrix} 0 & I_{3n} \\ -k_2 M_1 & -k_3 M_1 \end{bmatrix}$. Since the matrix $\bar{M}$ is Hurwitz, there exists $\bar{N} = \bar{N}^T > 0$ such that $\bar{M}^T\bar{N} + \bar{N}\bar{M} = -I_{6n}$. Define $\tilde{\varphi}_1 = \varphi_1$, $\tilde{\varphi}_2 = \text{sig}^{1/\alpha}(\varphi_2)$, $\tilde{\varphi} = [\tilde{\varphi}_1^T, \tilde{\varphi}_2^T]^T$, and the dilation $\delta_\lambda^r(\varphi_1, \varphi_2) = (\lambda\varphi_1^T, \lambda^\alpha\varphi_2^T)$. Consider the following Lyapunov function:

$$V_2(\varphi) = \tilde{\varphi}^T\bar{N}\tilde{\varphi} \quad (26)$$

where $\varphi = [\varphi_1^T, \varphi_2^T]^T$. We first focus on the case when $\epsilon_2 = 0$ and $\tilde{p} = 0$, such that (24) and (25) become

$$\dot{\varphi}_1 = k_1\varphi_2 \quad (27)$$

$$\dot{\varphi}_2 = -k_1 k_2 M_1 \text{sig}^{\alpha_1}(\varphi_1) - k_1^{\alpha_2} k_3 M_1 \text{sig}^{\alpha_2}(\varphi_2). \quad (28)$$

Let f_φ be the vector field of system defined by (27) and (28), and it can be verified that f_φ is homogeneous of degree $\alpha - 1$ with respect to the dilation $\delta_\lambda^r(\varphi_1, \varphi_2)$. Moreover, $V_2(\varphi)$ and $L_{f_\varphi}V_2(\varphi)$ are homogeneous of degree 2 and $\alpha + 1$ with respect to the dilation $\delta_\lambda^r(\varphi_1, \varphi_2)$, respectively. It follows from Lemma 4.2 in Bhat and Bernstein (2005) that $-c_5(\alpha, k_1)V_2^\beta(\varphi) \le L_{f_\varphi}V_2(\varphi) \le -c_6(\alpha, k_1)V_2^\beta(\varphi)$, where $c_5(\alpha, k_1) = -\min_{\{z:V_2(z)=1\}} L_{f_\varphi}V_2(z)$ and $c_6(\alpha, k_1) = -\max_{\{z:V_2(z)=1\}} L_{f_\varphi}V_2(z)$. Using the same method as that in Shen and Xia (2008) and Shen and Huang (2009), it can be verified that $\lim_{\alpha\to 1} c_6(\alpha, k_1) \ge k_1/\lambda_{\max}(\bar{N})$.

The time derivative of V_2 along with (24) and (25) is

$$\dot{V}_2 \le -c_6(\alpha, k_1)V_2^\beta(\varphi) + 2\tilde{\varphi}^T\bar{N}\begin{bmatrix} \theta^{1+\theta_1}M_1\epsilon_2 \\ \frac{1}{k_1\alpha}\text{diag}\left(|\varphi_2|^{\frac{1}{\alpha}-1}\right)M_1\tilde{p} \end{bmatrix}. \quad (29)$$

Define $\psi = [\psi_1^T, \psi_2^T, \cdots, \psi_n^T]^T$ and $\psi_i = \sum_{j=1}^{n} l_{ij}\tilde{p}_j + a_{i0}\tilde{p}_i (i = 1, 2, \cdots, n)$. Noticing that

$$\left\|\text{diag}\left(|\varphi_2|^{\frac{1}{\alpha}-1}\right)\psi\right\| \le \sum_{i=1}^{n}\sum_{j=1}^{3}|\varphi_{2i,j}|^{\frac{1}{\alpha}-1}|\psi_{ij}|$$

$$\frac{2\lambda_{\max}(\bar{N})}{k_1\alpha}|\varphi_{2i,j}|^{\frac{1}{\alpha}-1}|\psi_{ij}| \le \frac{1}{\sqrt{3n}}|\varphi_{2i,j}|^{\frac{1}{\alpha}} + c_{01}|\psi_{ij}|^{\frac{1}{\alpha}}$$

where $c_{01} = \frac{2\lambda_{\max}(\bar{N})}{k_1}\left(\frac{2\sqrt{3n}(1-\alpha)\lambda_{\max}(\bar{N})}{k_1\alpha}\right)^{1/\alpha-1}$ and Lemma 2 is applied, it follows that

$$\begin{aligned}\frac{2\lambda_{\max}(\bar{N})}{k_1\alpha}&\left\|\text{diag}\left(|\varphi_2|^{\frac{1}{\alpha}-1}\right)\psi\right\| \\ &\le \frac{1}{\sqrt{3n}}\left(\sum_{i=1}^{n}\sum_{j=1}^{3}|\varphi_{2i,j}|^{\frac{1}{\alpha}}\right) + c_{01}\left(\sum_{i=1}^{n}\sum_{j=1}^{3}|\psi_{ij}|^{\frac{1}{\alpha}}\right) \\ &\le \|\tilde{\varphi}_2\| + c_{01}\left(\sum_{i=1}^{n}\sum_{j=1}^{3}|\psi_{ij}|^{\frac{1}{\alpha}}\right)\end{aligned} \quad (30)$$

where Lemma 3 is applied. Thus, we obtain

$$\begin{aligned}\frac{2\lambda_{\max}(\bar{N})}{k_1\alpha}\|\tilde{\varphi}\|&\left\|\text{diag}\left(|\varphi_2|^{\frac{1}{\alpha}-1}\right)\psi\right\| \\ &\le 2\|\tilde{\varphi}\|^2 + \frac{c_{01}^2}{4}\left(\sum_{i=1}^{n}\sum_{j=1}^{3}|\psi_{ij}|^{\frac{1}{\alpha}}\right)^2 \\ &\le c_7 V_2 + c_8\sum_{i=1}^{n}\sum_{j=1}^{3}|\psi_{ij}|^{\frac{2}{\alpha}} \\ &= c_7 V_2 + c_8\psi^T\text{sig}^{\frac{2}{\alpha}-1}(\psi)\end{aligned} \quad (31)$$

where $c_7 = 2/\lambda_{\min}(\bar{N})$, $c_8 = 3nc_{01}^2/4$, and Lemma 5 is applied. Furthermore, noting that

$$\|\tilde{\varphi}\|\|\epsilon_2\| \leq \left(\sum_{i=1}^{2}\sum_{j=1}^{n}\sum_{m=1}^{3}|\tilde{\varphi}_{ij,m}|\right)\left(\sum_{j=1}^{n}\sum_{m=1}^{3}|\epsilon_{2j,m}|\right)$$

and

$$\begin{aligned}
&2\theta^{1+\theta_1}\lambda_{\max}(\bar{N})\lambda_{\max}(M_1)|\tilde{\varphi}_{ij,s}|\left(\sum_{j=1}^{n}\sum_{m=1}^{3}|\epsilon_{2j,m}|\right)\\
&\leq \frac{1}{(6n)^{1-\beta}}|\tilde{\varphi}_{ij,s}|^{1+\alpha} + \frac{\alpha c_{03}}{1+\alpha}\left(\sum_{j=1}^{n}\sum_{m=1}^{3}|\tilde{\epsilon}_{2j,m}|^{1+\alpha}\right)
\end{aligned}$$

where $s = 1,2,3$, $c_{02} = 2\theta^{1+\theta_1}\lambda_{\max}(\bar{N})\lambda_{\max}(M_1)$, $c_{03} = c_{02}\left[3nc_{02}(6n)^{1-\beta}/(1+\alpha)\right]^{1/\alpha}$, and Lemma 2 is applied, we obtain

$$\begin{aligned}
c_{02}\|\tilde{\varphi}\|\|\epsilon_2\| &\leq \frac{1}{(6n)^{1-\beta}}\left(\sum_{i=1}^{2}\sum_{j=1}^{n}\sum_{m=1}^{3}|\tilde{\varphi}_{ij,m}|^{1+\alpha}\right)\\
&\quad + \frac{6n\alpha c_{03}}{1+\alpha}\left(\sum_{j=1}^{n}\sum_{m=1}^{3}|\tilde{\epsilon}_{2j,m}|^{1+\alpha}\right)\\
&\leq \frac{1}{\lambda_{\min}^{\beta}(\bar{N})}V_2^{\beta} + \frac{6\times 3^{1-\beta}n\alpha c_{03}}{(1+\alpha)\lambda_{\min}^{\beta}(N)}\sum_{i=1}^{n}V_{oi}^{\beta}\\
&= c_9 V_2^{\beta} + c_{10}\sum_{i=1}^{n}V_{oi}^{\beta} \qquad (32)
\end{aligned}$$

where Lemma 3 is used. Applying the inequalities (31) and (32) to (29) leads to

$$\dot{V}_2 \leq -(c_6 - c_9)V_2^{\beta} + c_7 V_2 + c_8\psi^T \mathrm{sig}^{\frac{2}{\alpha}-1}(\psi) + c_{10}\sum_{i=1}^{n}V_{oi}^{\beta}. \qquad (33)$$

Remark 3. The control torque $\tau_i(i = 1,2,\cdots,n)$ given in (20) depends on the local information rather than the global information. Therefore, the control law is distributed. Furthermore, only attitude measurements are necessary for use in the control law, and finite-time stability of the overall closed-loop system can be achieved as will be shown in Theorem 2. Finally, the prior knowledge about the topology of information flow, i.e., the topology graph is connected and undirected, is necessary to implement the proposed control law.

Theorem 2. Consider that a group of spacecraft are described by (12) under an undirected connected communication graph, the finite-time observers are given by (13) and (16), and the distributed control law is defined by (20). For any given constant $V_M > 0$, if the initial conditions satisfy $\sum_{i=1}^{n}V_{oi}(\epsilon_i(0)) + V_1(\psi(0)) + V_2(\varphi(0)) \leq V_M$, where V_{oi}, V_1 and V_2 are defined by (14), (17) and (26), respectively, and Assumption 1 is satisfied, then there exist parameters $\theta, \alpha, k_1, k_2, k_3$, and $\beta_j(j = 1,2,3,4)$ such that the semi-global finite-time stability of the overall closed-loop system can be achieved.

Proof. Consider the following Lyapunov function:

$$V = K\sum_{i=1}^{n}V_{oi} + KV_1 + V_2 \qquad (34)$$

where $K > \max\{c_8, c_{10}\}$ is a large positive constant, V_{oi}, V_1 and V_2 are defined by (14), (17) and (26), respectively. The time derivative of V along with (15), (18) and (33) is

$$\begin{aligned}
\dot{V} &\leq c_7 V_2 - K\sum_{i=1}^{n}\left[(c_{2m}-1)V_{oi}^{\beta} - c_{3M}V_{oi}\right]\\
&\quad - K(\beta_3 - 1)\psi^T \mathrm{sig}^{\frac{2}{\alpha}-1}(\psi) - (c_6 - c_9)V_2^{\beta} \qquad (35)
\end{aligned}$$

where $c_{2m} = \min\{c_{2i}\}$ and $c_{3M} = \max\{c_{3i}\}(i = 1,2,\cdots,n)$. Note that the parameters θ and k_1 determine the magnitude of c_{2m} and c_6, respectively, we can choose parameters θ, β_3 and k_1 such that $c_{2m} - 1 > 0$, $\beta_3 - 1 > 0$, and $\bar{c}_6 = c_6 - c_9 > 0$. Then, we have

$$\begin{aligned}
\dot{V} &\leq -K\sum_{i=1}^{n}\left[(c_{2m}-1)V_{oi}^{\beta} - c_{3M}V_{oi}\right] - \bar{c}_6 V_2^{\beta} + c_7 V_2\\
&\leq -K\left[(c_{2m}-1)V_o^{\beta} - c_{3M}V_o\right] - \bar{c}_6 V_2^{\beta} + c_7 V_2\\
&= -KV_o^{\beta}\left[c_{2m} - 1 - c_{3M}V_o^{1-\beta}\right] - V_2^{\beta}(\bar{c}_6 - c_7 V_2^{1-\beta}) \qquad (36)
\end{aligned}$$

where $V_o(\epsilon) = \sum_{i=1}^{n}V_{oi}(\epsilon_i)$, and Lemma 3 is applied. Choosing θ and k_1 such that $c_{2m} > c_{3M}V_M^{1-\beta} + 1$ and $c_6 > c_7 V_M^{1-\beta} + c_9$, it follows that $\dot{V} < 0$ on $\bar{V} = V_o(\epsilon) + V_1(\psi) + V_2(\varphi) \leq V_M$. Thus, all signals in the closed-loop system are bounded, and there exists a positive constant Δ such that $\|q_i\| \leq \Delta, \|\hat{q}_i\| \leq \Delta, \|v_i\| \leq \Delta$, and $\|\hat{v}_i\| \leq \Delta$. From Lemma 6, if $\alpha \in (1/2, 1)$, then $\tilde{q}_i$ and $\tilde{v}_i(i = 1,2,\cdots,n)$ converge to zero in a finite time t_{f1}, and from Lemma 7, $\tilde{p}$ converges to zero in a finite time t_{f2}. When $t \geq \max\{t_{f1}, t_{f2}\}$, (29) becomes $\dot{V}_2 \leq -c_6(\alpha, k_1)V_2^{\beta}(\varphi)$, which indicates that χ_1 and $\hat{\chi}_2$ converge to zero in finite time as $\beta = (1+\alpha)/2 < 1$. Since $\chi_1 = M_1 e_1$, $\hat{\chi}_2 = M_1\hat{e}_2$, and $e_2 = \tilde{v} + \hat{e}_2$, we obtain that the tracking errors e_1 and e_2 converge to zero in finite time. □

Remark 4. If $\alpha = 1$, the observer (13) becomes:

$$\begin{cases} \dot{\hat{q}}_i = \hat{v}_i + \theta\beta_1\tilde{q}_i \\ \dot{\hat{v}}_i = f_i(q_i, \hat{v}_i) + g_i(q_i)\tau_i + \theta^2\beta_2\tilde{q}_i \end{cases} \qquad (37)$$

and the control law (20) reduces to:

$$\tau_i = g_i^{-1}[-k_1^2 k_2 \chi_{1i} - k_1 k_3 \hat{\chi}_{2i} - f_i(q_i, \hat{v}_i) - \theta^2 \beta_2 \tilde{q}_i + p_i], \quad i = 1, 2, \cdots, n. \tag{38}$$

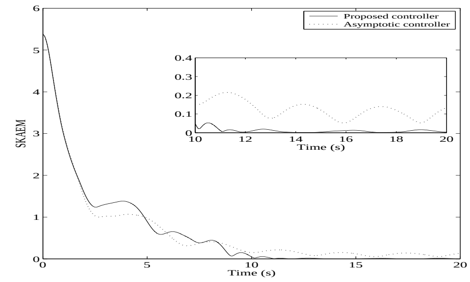
According to the proof procedure of Theorem 2, it is easy to verify that e_1 and e_2 converge to zero asymptotically.

4 Simulation Results

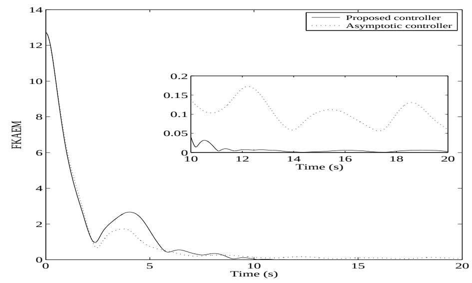
In this section, numerical simulations are presented to verify the effectiveness of the proposed controller. A scenario where there are six spacecraft and one virtual leader is considered. The mass moment of inertia tensor for each spacecraft is chosen to be the same as in Table 1 in Ren (2007). The weighted adjacency matrix $A = (a_{ij})_{6\times 6}$ is taken as $a_{12} = a_{21} = 0.4$, $a_{16} = a_{61} = 0.6$, $a_{23} = a_{32} = 0.6$, $a_{34} = a_{43} = 0.6$, $a_{45} = a_{54} = 0.8$, $a_{56} = a_{65} = 0.6$, and $a_{ij} = 0$ for other elements. The adjacency matrix B is chosen as $B = \mathrm{diag}[0.5, 0, 0, 0, 0, 0.5]$. The reference attitude is $q_0 = 0.2[\cos(0.2t), \sin(0.2t), \sqrt{3}]^T$. The initial attitude of each spacecraft is considered to be: $q_1(0) = [0, 1, \sqrt{3}]^T, q_2(0) = -0.4[1, 1, \sqrt{2}]^T, q_3(0) = 1.4[\sqrt{3}, 1, 0]^T, q_4(0) = -0.6[\sqrt{3}, 0, 1]^T, q_5(0) = 1.5[1, \sqrt{2}, 1]^T$, and $q_6(0) = -1.2[1, 1, \sqrt{2}]^T$. The initial angular velocity is $\omega_i(0) = 0 (i = 1, 2, \cdots, 6)$. The limit on the control torque is considered to be $|\tau_{ij}| \le 2$Nm, where $j = 1, 2, 3$. The observer and controller parameters are set as: $\alpha = 0.8, \theta = 10, \beta_1 = \beta_2 = 2, \beta_3 = 4, \beta_4 = 0.5, k_1 = 3, k_2 = 1$, and $k_3 = 2$. The external disturbances are assumed as $\vartheta_i = 0.1[\sin(it), \cos(it), \sin(2it)]^T$. In this case, the attitude dynamics given in (10) becomes $J_i \dot{\omega}_i = -\omega_i^\times J_i \omega_i + \tau_i + \vartheta_i$.

The performance comparison between the finite-time controller (20) and the asymptotic controller (38) is examined. For comparison, the station-keeping attitude error metric (SKAEM) and the formation-keeping attitude error metric (FKAEM) are used, which are respectively defined as SKAEM $= \sqrt{\sum_{i=1}^{6} \|e_{1i}\|^2}$, and FKAEM $= \sqrt{\sum_{i=1}^{6}\sum_{j=i+1}^{6} \|q_i - q_j\|^2}$. Another metric to measure the overall control effort (OCEM) exerted by the spacecraft is given by OCEM $= \sqrt{\sum_{i=1}^{6} \|\tau_i\|^2}$.

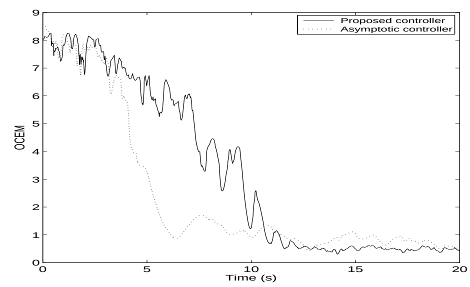
The response of SKAEM, FKAEM and OCEM for the finite-time controller (20) and the asymptotic controller (38) is shown in Fig. 1. It is found that the proposed controller can provide faster convergence and higher attitude coordination performance than the asymptotic controller, which indicates that the proposed finite-time controller can provide a better disturbance rejection property than the asymptotic controller (38). For the transient phase, it is seen from Fig. 1 (c) that the proposed controller requires a slightly larger control effort than the controller (38). This can be explained as follows: for any $0 < |x| < 1$, we have $|x| < |x|^\alpha$ with $0 < \alpha < 1$, which results in a larger control input (20) than (38) during the transient phase. However, since the finite-time control law can provide a faster transient response and a higher accuracy control performance, during the steady-state stage, it is observed from Fig. 1 (c) that the proposed controller can reduce the fuel consumption as compared with the controller (38).



(a) SKAEM



(b) FKAEM



(c) OCEM

Fig. 1. Performance comparison between the finite-time controller (20) and the asymptotic controller (38).

5 Conclusions

Based on a finite-time observer, a decentralized finite-time observer, and the homogenous method, a distributed velocity-free finite-time attitude coordination control law was developed for spacecraft formations. The performance of the proposed controller was compared with an asymptotic control law. It was shown that the proposed controller can provide a faster convergence rate, better disturbance rejection property, and higher attitude coordination accuracy than the asymptotic controller. By removing the requirement of angular velocity measurements, the cost related to sensors can be reduced.

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